Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				

Examiner Comments	Total Marks

MATHEMATICS

AS LEVEL PAPER 1

CM

Silver Set A

Time allowed: 2 hours

Instructions to candidates:

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer **all** of the questions.
- Write your answer for each question in the spaces provided.
- You should show sufficient workings to make your methods clear.
- Answers without working may not gain full credit.
- Answers should be given to three significant figures unless stated otherwise.
- You may use a calculator.

Information to candidates:

- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.

Advice to candidates:

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1 Given that

$$3^x \times 9^{x+2} = \frac{1}{27}$$

find the value of x.

(3)

Question 1 continued

TOTAL 3 MARKS

2

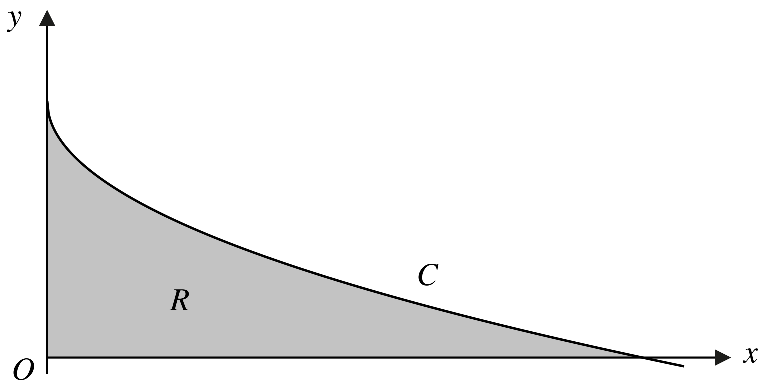

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 3 - \sqrt{x}$, $x \geqslant 0$.

The region R, shown shaded in Figure 1, is bounded by C and the x axis.

Showing your working clearly, find the area of R.

(5)

Question 2 continued

TOTAL 5 MARKS

Do not write outside the box

3 $$f(x) = 2x^3 + 12x^2 + ax + a$$

where a is a constant.

Given that $(x + 3)$ is a factor of $f(x)$,

(a) show that $a = 27$.

(1)

(b) Deduce that the equation $f(x) = 0$ has only one real root.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

Question 3 continued

TOTAL 5 MARKS

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4

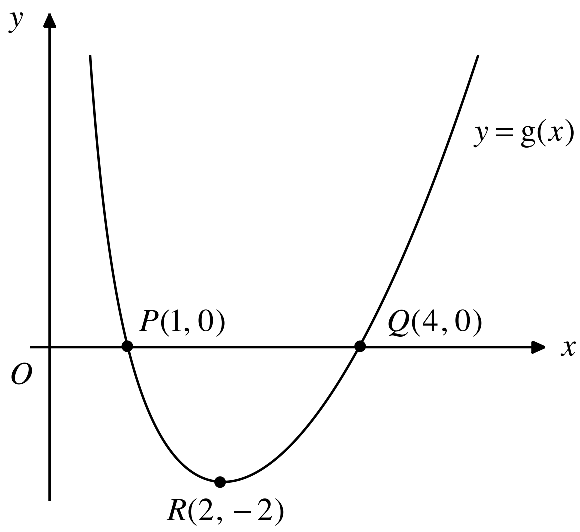

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{g}(x)$.

The curve meets the x axis at the points $P(1, 0)$ and $Q(4, 0)$.
The point $R(2, -2)$ is a turning point on the curve.
The y axis is an asymptote to the curve.

(a) State the roots of $\mathrm{g}(2x) = 0$.

(1)

(b) Write down the equation of the asymptote to the curve $y = \mathrm{g}(x - 3)$.

(1)

(c) Using set notation, write down the range of values of k for which the equation

$$4\,\mathrm{g}(x) = k$$

has two real roots.

(1)

(d) Sketch the curve with equation $y = \mathrm{g}'(x)$, showing clearly the coordinates of any intersections with the coordinate axes.

(2)

Question 4 continued

TOTAL 5 MARKS

5 The circle C has the equation

$$x^2 + y^2 + px + qy = 14$$

where p and q are constants.

Given that C has centre $(2, -5)$, find the radius of the circle.

Show your working clearly.

(5)

Question 5 continued

TOTAL 5 MARKS

6 A tank is initially completely filled with liquid. An outlet is opened at the bottom of the tank and the liquid begins to drain from the tank.

At time t minutes after the outlet is opened, the amount of liquid in the tank is V cm^3.

Kyle creates a model for the liquid flow out of the tank. The model includes the following assumptions:

- the initial volume of liquid in the tank is 300 cm^3,
- it takes two minutes for the volume of liquid in the tank to reach 80 cm^3,
- the rate of flow of liquid out of the tank is proportional to the amount of liquid in the tank.

(a) Using Kyle's model, find an expression for V in terms of t.

(5)

(b) Explain why Kyle's model should **not** be used to predict the time taken for the container to empty.

(1)

Question 6 continued

TOTAL 6 MARKS

7 Using algebra, solve the equation

$$(2+3x)^5+(2-3x)^5=244$$

Show your working clearly.

(5)

Question 7 continued

TOTAL 5 MARKS

8

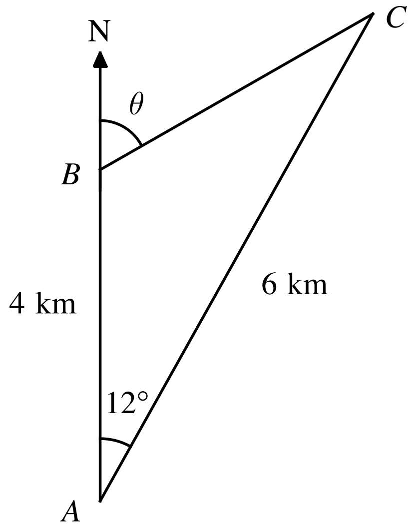

Figure 2

Figure 2 shows the positions of planes A, B and C at a particular instant in time, which are assumed to be in the same horizontal plane. Plane B is 4 km due north of plane A and plane C is 6 km from plane A.

(a) Calculate the distance between plane B and plane C.
Give your answer to the nearest 0.5 km.

(2)

The bearing of plane C from plane B is $\theta°$, as shown in Figure 2.

(b) Find the value of θ to the nearest degree.

(3)

Question 8 continued

TOTAL 5 MARKS

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9 A student was set the following problem.

"Find the values of x that satisfy $\dfrac{5-x}{x} \leqslant 6$."

The student's attempt is shown below.

$$\frac{5-x}{x} \leqslant 6$$

$$x(5-x) \leqslant 6x^2 \longleftarrow \text{Line 2}$$

$$5x - x^2 \leqslant 6x^2$$

$$7x^2 - 5x \leqslant 0$$

$$x(7x-5) \leqslant 0$$

$$0 \leqslant x \leqslant \frac{5}{7}$$

In line 2, the student correctly multiplies both sides by x^2.

(a) Suggest why the student has multiplied both sides by x^2 rather than x.

(1)

The rest of the student's solution contains two errors.

(b) Identify these two errors made by the student.

(2)

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Question 9 continued

TOTAL 3 MARKS

10 A curve C is such that

$$\frac{d^2 y}{dx^2} = 12x - 4$$

Given that C has a stationary point at $(2, -1)$,

(a) show in clear stages that

$$y = 2x^3 - 2x^2 - 16x + k$$

where k is a constant to be found.

(6)

(b) Find the coordinates of the other stationary point on C **and** determine its nature.

(4)

Question 10 continued

Question 10 continued

Question 10 continued

TOTAL 10 MARKS

11 (a) Prove that for all positive values of x and y

$$\frac{1}{2}(x+y) \geqslant \sqrt{xy}$$

(2)

Rectangle R has perimeter P and area A.

(b) Show that $P \geqslant 4\sqrt{A}$.

(3)

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Question 11 continued

TOTAL 5 MARKS

12 A group of biologists did an experiment with a bacterial colony.

A model to represent how the size of the colony varied during the experiment is given by

$$N = 112 - 0.4(t - 12)^2, \quad t \geqslant 0$$

where N is the number of bacteria in the colony, in thousands, at time t hours after the experiment started.

(a) Find the initial number of bacteria in the colony.
(1)

(b) Calculate the change in the number of bacteria in the colony during the third hour of the experiment.
(2)

During the experiment, the biologists add an antibiotic to the colony that causes the population of the colony to decrease to 0.

Theo looks at the model and assumes that the biologists add the antibiotic when $t = 12$.

(c) Explain, in relation to the model, the significance of this value of t.
(1)

(d) Using Theo's assumption, find the time taken for the antibiotic to eliminate the colony.
(3)

(e) Suggest **one** reason why Theo's assumption may not be correct.
(1)

Question 12 continued

TOTAL 8 MARKS

13 (a) Prove that

$$\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}, \qquad x \neq 90°(2n - 1),\ n \in \mathbb{Z}$$

(4)

(b) Hence, for $-180° \leqslant x \leqslant 180°$, solve the equation

$$\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{5}$$

giving your answers to two decimal places.

(5)

Question 13 continued

Question 13 continued

Question 13 continued

TOTAL 9 MARKS

14

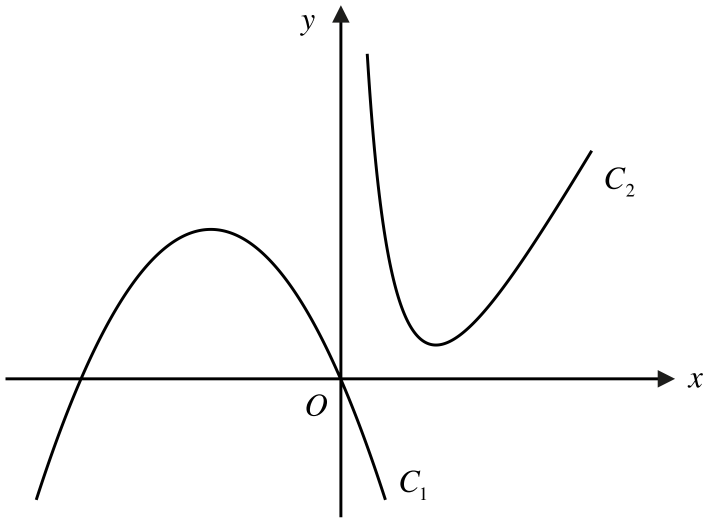

Figure 3

The curve C_1, shown in Figure 3, has the equation $y = -x^2 - 3x$.

The point $P\left(-\frac{1}{2}, \frac{5}{4}\right)$ lies on C_1.

The curve C_2, also shown in Figure 3, has the equation $y = 3\ln^2 x - \frac{3}{2}\ln x + \frac{1}{2}x$.

The normal to C_1 at the point P intersects C_2 at the points Q and R.

(a) Show that the x coordinates of the points Q and R satisfy

$$2\ln^2 x - \ln x - 1 = 0$$

(5)

(b) Hence find the exact coordinates of Q and R.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

Question 14 continued

Question 14 continued

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Question 14 continued

TOTAL 9 MARKS

15 Relative to a fixed origin O, the points A, B and C are such that

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ p \end{pmatrix}, \ \overrightarrow{OB} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} \text{ and } \overrightarrow{OC} = \begin{pmatrix} 1 \\ -9 \end{pmatrix}, \text{ where } p \text{ is a positive constant}$$

(a) Find, in terms of p, expressions for $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

(2)

The point D is such that $ABCD$ is a parallelogram.

Given that the perimeter of $ABCD$ is 30 units,

(b) determine the value of p.

(4)

(c) Hence find the position vector of D.

(2)

Question 15 continued

Question 15 continued

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Question 15 continued

TOTAL 8 MARKS

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16

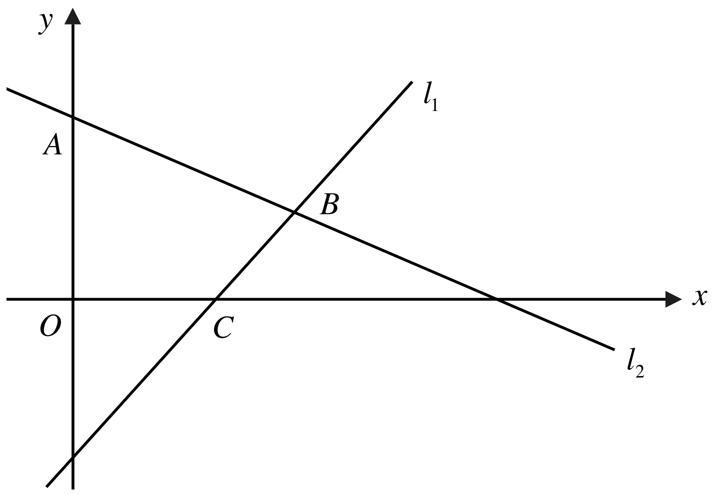

Figure 4

The line l_1 has the equation $3x - y - 4 = 0$.

The line l_2 is perpendicular to l_1 and passes through the point (3, 1).

The point A is where l_2 meets the y axis.
The point B is where l_1 and l_2 intersect.
The point C is where l_1 meets the x axis.

Figure 4 shows the lines l_1 and l_2 and the points A, B and C.

Find the area of the quadrilateral $OABC$, where O is the origin.
Show all of your working.

(9)

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Question 16 continued

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Question 16 continued

Question 16 continued

END OF PAPER **TOTAL 9 MARKS**

TOTAL FOR PAPER IS 100 MARKS