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# AS Level Maths

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Silver Set A, Paper 1 (Edexcel)

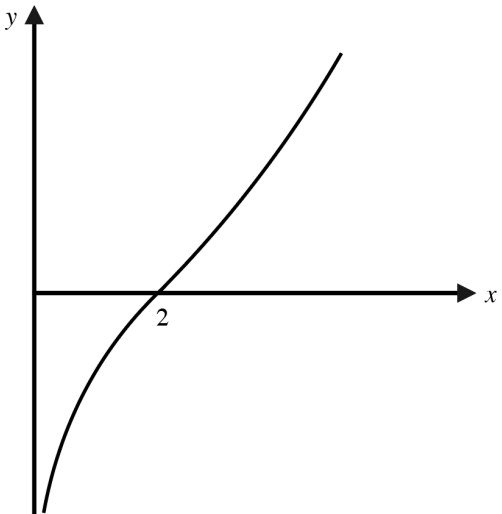
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AS Level Maths – CM Paper 1 (for Edexcel) / Silver Set A

Question	Solution	Partial Marks	Guidance
<b>1</b>	$3^x \times 3^{2(x+2)} = 3^{-3}$ $x + 2(x+2) = -3 \Rightarrow x = \dots$ $x = -\frac{7}{3}$	B1 M1 A1 oe [3]	Writes a correct equation in powers of 3 only Uses correct index laws to form a linear equation in $x$ and then solves for $x$ Correct value of $x$
<b>1 ALT</b>	$\log(3^x \times 9^{x+2}) = \log \frac{1}{27}$ $\log 3^x + \log 9^{x+2} = \log \frac{1}{27}$ $x \log 3 + (x+2) \log 9 = \log \frac{1}{27} \Rightarrow x = \dots$ $x = \frac{\log \frac{1}{27} - 2 \log 9}{\log 3 + \log 9} \left\{ \Rightarrow x = -\frac{7}{3} \right\}$	B1 M1 A1 oe [3]	Writes a correct equation involving logarithms Uses correct rules of logarithms to form a linear equation in $x$ and then solves for $x$ Correct value of $x$ – accept the answer given in terms of logs ISW once a correct answer is reached
<p><b><u>Special cases for Question 1:</u></b></p> <p>After 0/3 scored, then allow SCB1 for any of the following seen</p> <ul style="list-style-type: none"> <li>• <math>3^x \times 9^{x+2} \rightarrow 3^{x+2(x+2)}</math></li> <li>• <math>3^x \times 9^{x+2} \rightarrow 9^{\frac{1}{2}x+x+2}</math></li> <li>• <math>\log(3^x \times 9^{x+2}) \rightarrow x \log 3 + (x+2) \log 9</math></li> <li>• <math>\log(3^x \times 9^{x+2}) \rightarrow x \log 3 + 2(x+2) \log 3</math> (any log base)</li> </ul>			

2	$3 - \sqrt{x} = 0 \Rightarrow x = 9$ $\int_0^9 (3 - \sqrt{x}) dx = \left[ 3x - \frac{2x^{\frac{3}{2}}}{3} \right]_0^9$ $= 3(9) - \frac{2}{3}(9)^{\frac{3}{2}} - 3(0) + \frac{2}{3}(0)$ $= 9$	M1* A1 B1 M1(dep*) A1 cso <b>[5]</b>	Attempts to integrate at least one of the terms indefinitely M1 for $x^n \rightarrow x^{n+1}$ Correct indefinite integration  For correct limits seen on the integral or seen used Correct method to evaluate an integral between $x = 0$ and $x = k > 0$ Must see substitution if they are working with incorrect limits or an incorrect indefinite integral Obtains correct final answer
3 (a)	$2(-3)^3 + 12(-3)^2 + a(-3) + a = 0$ $\Rightarrow -54 + 108 - 3a + a = 0$ $\Rightarrow 2a = 54$ $\Rightarrow a = 27 \quad \mathbf{AG}$	B1  <b>[1]</b>	Shows the result convincingly Need to see the substitution or candidates state they are using $f(-3) = 0$ May also proceed to long division in terms of $a$ and set the remainder = 0
3 (b)	$2x^3 + 12x^2 + 27x + 27 = (x + 3)(2x^2 + 6x + 9)$ Discriminant of quadratic factor is $6^2 - 4(2)(9) = -36$  Since $-36 < 0$ , the quadratic factor has no real roots $6^2$ Thus $f(x) = 0$ only has one real root (which is $x = -3$ )	M1*  A1 M1(dep*)  A1  <b>[4]</b>	Complete method to find the quadratic factor <ul style="list-style-type: none"> <li>If using long division, they must complete the division in full and obtain <math>2x^2 + px + q</math>, <math>p, q \neq 0</math> – only condone slips in arithmetic, not method</li> <li>If using inspection, M1 for two coefficients correct</li> </ul> Must be working from the correct cubic Obtains correct quadratic factor Begins method to show their quadratic factor has no real roots, eg by evaluating the discriminant, completing the square or finding $x$ coordinate of turning point by calculus M0 if their quadratic discriminant has non-negative discriminant or  Shows the result convincingly with conclusion eg ‘thus only one real root’, ‘as required’, ‘qed’

4 (a)	$x = 1/2, x = 2$	B1 oe [1]	Correct roots oe B0 for (1/2, 0) and (2, 0)
4 (b)	$x = 3$	B1 oe [1]	Correct equation of the asymptote oe
4 (c)	$\{k \in \mathbb{R} : k > -8\}$	B1 oe [1]	Correct set of values of $k$ Allow omission of ' $\in \mathbb{R}$ ' and accept equivalent sets
4 (d)	<p><b>EXAMPLE (see notes):</b></p> 	B1  B1  [2]	<p>Correct shape – the graph needs to be concave for <math>x &lt; “2”</math> and convex for <math>x &gt; “2”</math>. Be flexible in the level of convexity as this will depend on how candidates interpret the diagram. Do not allow a straight line for all <math>x</math> but <u>allow one</u> of the segments (<math>&lt; 2</math> or <math>&gt; 2</math>) to be a straight line</p> <p>Correct intersection with the <math>x</math> axis (and no others)</p>

<p><b>5</b></p>	$\left(x + \frac{p}{2}\right)^2 - \frac{p^2}{4} + \left(y + \frac{q}{2}\right) - \frac{q^2}{4} = 14$ $\left(x + \frac{p}{2}\right)^2 + \left(y + \frac{q}{2}\right) = 14 + \frac{p^2}{4} + \frac{q^2}{4}$ <p>Centre is (2, -5), so</p> $\frac{p}{2} = -2 \Rightarrow p = -4, \quad \frac{q}{2} = 5 \Rightarrow q = 10$ $r^2 = 14 + \frac{(-4)^2}{4} + \frac{(10)^2}{4} = 43$ $r = \sqrt{43}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p><b>[5]</b></p>	<p>Correctly completes the square on the <math>x</math> and <math>y</math> terms All terms need not be seen together and can be implied</p> <p>Complete method to find <math>p</math> and <math>q</math> by comparing with their <math>p/2</math> and their <math>q/2</math> (NB M0M1 is possible if constants not seen) Allow sign confusion for the M1</p> <p>Correct values of <math>p</math> and <math>q</math> (correct values imply 3/3)</p> <p>Uses their value for <math>p</math> and <math>q</math> to find the <u>radius</u> of the circle Correct radius Decimal equivalent of 6.24 is A1 (must be correct to 3 sf)</p>
<p><b>5</b> <b>ALT</b></p>	$(x - 2)^2 + (y + 5)^2 = r^2$ $x^2 - 4x + 4 + y^2 + 10y + 25 = r^2$ $x^2 + y^2 - 4x + 10y = r^2 - 29$ <p>So <math>p = -4</math> and <math>q = 10</math></p> $r^2 - 29 = 14 \Rightarrow r^2 = 43$ $\Rightarrow r = \sqrt{43}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p><b>[5]</b></p>	<p>Writes correct form for the equation of the circle Allow any symbol on the RHS – in particular, do not penalise candidates that think the RHS <i>is</i> the radius (this is assessed later)</p> <p>Attempts to expand the brackets and compares coefficients to find <math>p</math> and <math>q</math>. Their LHS must be of the form <math>(x \pm 4)^2 + (y \pm 6)^2</math> Correct values of <math>p</math> and <math>q</math> (correct values imply 3/3)</p> <p>Uses their value for <math>p</math> and <math>q</math> to find the <u>radius</u> of the circle Correct radius Decimal equivalent of 7.87 is A1 (must be correct to 3 sf)</p>

<p><b>6 (a)</b></p>	$V = Ae^{kt}$ <p>Initial volume = 300, so <math>A = 300</math></p> $80 = 300e^{2k} \Rightarrow e^{2k} = \frac{8}{30}$ $\Rightarrow 2k = \ln \frac{8}{30}$ $\Rightarrow k = \frac{1}{2} \ln \frac{8}{30} \quad (= -0.6608\dots)$ <p>So <math>V = 300e^{\left(\frac{1}{2} \ln \frac{8}{30}\right)t}</math></p>	<p>M1* A1</p> <p>M1(dep*)</p> <p>M1</p> <p>A1</p> <p><b>[5]</b></p>	<p>States or implies use of an exponential model Obtains <math>V = 300e^{kt}</math> (or implied) (note this implies M1A1)</p> <p>Uses <math>V(2) = 80</math> to form an equation in their <math>k</math> and re-arranges for <math>e^{2k}</math> Their <math>A</math> in the model must be numerical</p> <p>Takes logs and re-arranges for their <math>k</math></p> <p>Obtains <math>V</math> in terms of <math>t</math> Accept decimal equivalents in numerator correct to 3 sf</p>
<p><b>6 (b)</b></p>	<p>eg volume does not actually reach zero according to the model / it would take infinite time</p>	<p>B1</p> <p><b>[1]</b></p>	<p>Correct explanation for why the model is inappropriate</p>
<p><b>7</b></p>	<p>Odd powers of <math>x</math> cancel, so</p> $(2 + 3x)^5 + (2 - 3x)^5 = 2[2^5 + {}^5C_2(2)^3(3x)^2 + {}^5C_4(2)^1(3x)^4]$ $= 64 + 1440x^2 + 1620x^4$ <p>Hence</p> $(2 + 3x)^5 + (2 - 3x)^5 = 244$ $\Rightarrow 1620x^4 + 1440x^2 - 180 = 0$ $\Rightarrow 9x^4 + 8x^2 - 1 = 0$ $\Rightarrow (9x^2 - 1)(x^2 + 1) = 0$ $\Rightarrow x^2 = \frac{1}{9}$ $\Rightarrow x = \pm \frac{1}{3}$	<p>M1</p> <p>M1*</p> <p>A1 oe</p> <p>M1(dep*)</p> <p>A1</p> <p><b>[5]</b></p>	<p>One term of the form <math>{}^5C_k(2)^k(\pm 3x)^{5-k}</math>, <math>k \neq 0, 5</math> (powers may be interchanged) <b>OR</b> correct unsimplified expansion of <math>(2 \pm 3x)^3</math> seen</p> <p>Complete method to expand both brackets and find their sum NB <math>2^5 + {}^5C_2(2)^3(3x)^2 + {}^5C_4(2)^1(3x)^4</math> is M1 until multiplied by 2 NB M0M1 is possible</p> <p>Obtains correct three term quartic (or equivalent)</p> <p>Complete method to solve a three-term quartic for <math>x</math> NB solving for <math>x^2</math> is not enough Obtains correct values of <math>x</math> <b><u>Both correct values only with verification seen can score SCB1B1 (unless they go on to show these are the only solutions)</u></b></p>

<p><b>8 (a)</b></p>	$BC^2 = 4^2 + 6^2 - 2(4)(6)\cos(12)$ $\Rightarrow BC^2 = 5.0489\dots$ $\Rightarrow BC = \sqrt{5.0489\dots}$ $\Rightarrow BC = 2.246\dots$ <p>(so distance between <math>B</math> and <math>C</math> is) <b>2.0</b> km (nearest 0.5 km)</p>	<p>M1</p> <p>A1 cao</p> <p>[2]</p>	<p>Substitutes values correctly into the cosine rule to form equation in <math>BC^2</math></p> <p>NB allow any variable on the LHS, but...</p> $x^2 = 4^2 + 6^2 - 2(4)(6)\cos(12)$ is M1 $x = 4^2 + 6^2 - 2(4)(6)\cos(12)$ is M0 <u>until we see them sqrt</u> <p>because there is no evidence they are using the correct cosine rule</p> <p>Correct distance between the planes</p> <p>Units not necessary</p>
<p><b>8 (b)</b></p>	$\frac{2.246}{\sin 12} = \frac{6}{\sin ABC} \Rightarrow \sin ABC = \dots$ $\Rightarrow \sin ABC = \frac{6 \sin 12}{2.246}$ <p>Since <math>ABC</math> is obtuse, <math>ABC = 180 - \sin^{-1}\left(\frac{6 \sin 12}{2.246}\right)</math></p> $= 146.27\dots$ <p>So <math>\theta = 180 - 146.27\dots =</math> awrt <b>34</b></p>	<p>M1</p> <p>M1</p> <p>A1 cao</p> <p>[3]</p>	<p>Uses correct sine rule to form equation involving angle <math>ABC</math> and re-arranges for <math>\sin ABC</math></p> <p>NB no marks if they find <math>ACB</math> until they involve <math>ABC</math></p> <p>Complete method to find <math>\theta</math></p> <p>Correct value of <math>\theta</math> to nearest degree</p>
<p><b>8 (b)</b> <b>ALT</b></p>	$\cos ABC = \frac{4^2 + 2.246^2 - 6^2}{2(4)(2.246)}$ $\Rightarrow \cos ABC = -0.8317\dots$ $\Rightarrow ABC = \cos^{-1}(-0.8317\dots) = 146.27\dots$ <p>So <math>\theta = 180 - 146.27\dots =</math> awrt <b>34</b></p>	<p>M1</p> <p>M1</p> <p>A1 cao</p> <p>[3]</p>	<p>Uses correct cosine rule to form an equation involving angle <math>ABC</math></p> <p>NB no marks if they find <math>ACB</math> until they involve <math>ABC</math></p> <p>Complete method to find <math>\theta</math></p> <p>Correct value of <math>\theta</math> to nearest degree</p>

9 (a)	eg we know $x^2$ is positive but $x$ may not be positive	B1 [1]	A correct suggestion
9 (b)	eg in $7x^2 - 5x \leq 0$ , the inequality sign should be $\geq$ eg $x = 0$ should not be included in the final solution	B1 B1 [2]	Identification of the first error (in line 4) <i>Allow the direction/sign of the inequality is incorrect (or better)</i> Identification of the second error (in line 6)
10 (a)	$\frac{dy}{dx} = \int (12x - 4) dx$ $= 6x^2 - 4x + c$ <p>When <math>x = 2, \frac{dy}{dx} = 0,</math>  <math>\Rightarrow 0 = 6(2)^2 - 4(2) + c</math>  <math>\Rightarrow c = -16</math></p> <p>So <math>\frac{dy}{dx} = 6x^2 - 4x - 16</math></p> $y = \int (6x^2 - 4x - 16) dx$ $= 2x^3 - 2x^2 - 16x + k$ <p>When <math>x = 2, y = -1,</math>  <math>\Rightarrow -1 = 2(2)^3 - 2(2)^2 - 16(2) + k</math>  <math>\Rightarrow k = 23</math></p> <p>So <math>y = 2x^3 - 2x^2 - 16x + 23</math></p>	M1*  M1(dep*)  A1  M1**(dep*)  M1(dep**)  A1 cso [6]	Attempts to integrate to find $dy/dx$ Need to see $x^n \rightarrow x^{n+1}$ <b>and</b> a constant of integration included M0 if their LHS is $y$  Attempts to use the correct initial conditions to evaluate their constant of integration <i>NB if initial conditions used as limits in an integral, then the method to evaluate the limits must be correct</i>  Obtains correct expression for $dy/dx$ A1 may be awarded once $c = -16$ is seen  Integrates their $dy/dx$ to find an expression for $y$ Must be working from $px^2 + qx + r, pqr \neq 0$ , with <u>all constants numerical</u> For the M1, looking for use of $x^n \rightarrow x^{n+1}$ <b>and</b> a constant of integration included. Allow same letter for constant as in $dy/dx$ if used correctly  Attempts to use the correct initial conditions to evaluate a constant of integration <i>Blue comment in the 2<sup>nd</sup> M1 also applies here</i> Shows the result convincingly <u>writing <math>y</math> in terms of <math>x</math></u> <b>OR</b> $y = 2x^3 - 2x^2 - 16x + k$ , followed by $k = 23$ (with working) <b>and</b> a conclusion (eg “as required”, “hence shown”, “qed”, etc) is A1



<p><b>10 (b)</b></p>	$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 4x - 16 = 0$ $\Rightarrow 3x^2 - 2x - 8 = 0$ $\Rightarrow (x-2)(3x+4) = 0$ <p>So other stationary point has x coordinate <math>-\frac{4}{3}</math></p> <p>Then <math>y = 2\left(-\frac{4}{3}\right)^3 + 2\left(-\frac{4}{3}\right)^2 - 16\left(-\frac{4}{3}\right) + 23 = \frac{973}{27}</math></p> <p>So other stationary point has coordinates <math>\left(-\frac{4}{3}, \frac{973}{27}\right)</math></p> <p>Now <math>\left.\frac{d^2y}{dx^2}\right _{x=-\frac{4}{3}} = 12\left(-\frac{4}{3}\right) - 4 = -20</math></p> <p>Since the second derivative is negative at <math>x = -4/3</math>, the point is a maximum point</p>	<p>M1</p> <p>A1 oe</p> <p>M1</p> <p>A1 cso</p> <p>[4]</p>	<p>Attempts to solve <math>6x^2 - 4x - 16 = 0</math> for <math>x</math> <b>and</b> then substitute their found x coordinate into their expression for <math>y</math> <u>Must be working from the correct derivative</u></p> <p>Obtains correct coordinates of the other stationary point Accept decimal equivalents that are correct to 3 sf</p> <p>Attempts to find the nature of <u>their</u> second stationary point, eg testing derivative on either side or evaluating the <u>sign</u> of the second derivative (note the actual value is not required) Allow any value of <math>x</math> used except <math>x = 2</math></p> <p>States that the point is a maximum with justification CSO</p>
<p><b>11 (a)</b></p>	<p>(since <math>x, y &gt; 0</math>, we have)</p> $(\sqrt{x} - \sqrt{y})^2 \geq 0$ $\Rightarrow x - 2\sqrt{xy} + y \geq 0$ $\Rightarrow x + y \geq 2\sqrt{xy}$ $\Rightarrow \frac{1}{2}(x + y) \geq \sqrt{xy} \quad \mathbf{AG}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Starts with <math>(\sqrt{x} - \sqrt{y})^2 \geq 0</math> and expands the brackets ALT: may start from <math>(x - y)^2 \geq 0</math>. In this case, need to see square rooting for the M1 or use of a substitution</p> <p>Obtains the correct result convincingly with no errors</p>

<p><b>11 (b)</b></p>	<p>Let <math>x</math> be the width and <math>y</math> the length of <math>R</math> Then <math>P = 2x + 2y</math> and <math>A = xy</math></p> <p>So <math>x + y = \frac{1}{2}P</math> and <math>\sqrt{xy} = \sqrt{A}</math></p> <p>Then using (a), <math>\frac{1}{4}P \leq \sqrt{A} \Rightarrow P \leq 4\sqrt{A}</math></p>	<p>M1*</p> <p>M1(dep*)</p> <p>A1</p> <p>[3]</p>	<p>Sets up the problem by forming expressions for the perimeter and area of the rectangle May use any symbol for the width and length</p> <p>Attempts to relate their <math>P</math> and <math>A</math> to the inequality in (a) eg by making a correct relation of the perimeter to <math>x + y</math> and the area to <math>\sqrt{xy}</math></p> <p>Shows the result convincingly with no errors</p>
<p><b>11 (b)</b> <b>ALT</b></p>	<p>Let <math>x</math> be the width and <math>y</math> the length of <math>R</math> Then <math>P = 2x + 2y</math> and <math>A = xy</math></p> <p>Using part (a), <math>\frac{1}{2} \left[ \frac{1}{2}(2x + 2y) \right] \geq \sqrt{xy}</math></p> <p><math>\Rightarrow \frac{1}{4}P \leq \sqrt{A}</math> <math>\Rightarrow P \leq 4\sqrt{A}</math></p>	<p>M1*</p> <p>M1(dep*)</p> <p>A1</p> <p>[3]</p>	<p>Sets up the problem by forming expressions for the perimeter and area of the rectangle May use any symbol for the width and length</p> <p>Attempts to relate their <math>P</math> and <math>A</math> to the inequality in (a) eg “introducing” <math>P</math> onto the LHS or multiplying both sides by 4</p> <p>Shows the result convincingly with no errors</p>
<p><b>12 (a)</b></p>	<p>54400</p>	<p>B1</p> <p>[1]</p>	<p>Cao</p>
<p><b>12 (b)</b></p>	<p><math>N(3) = 112 - 0.4(3 - 12)^2 = 79600</math> <math>N(2) = 112 - 0.4(2 - 12)^2 = 72000</math></p> <p>So change in population size = <math>79600 - 72000 = 7600</math></p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Complete method to find change in the population size in the third hour of the experiment</p> <p>Correct change in the population size</p>
<p><b>12 (c)</b></p>	<p>eg (corresponds to) the time at which the population is largest</p>	<p>B1</p> <p>[1]</p>	<p>Explains the significance of this time</p>

<p><b>12 (d)</b></p>	$112 - 0.4(t - 12)^2 = 0$ $\Rightarrow (t - 12)^2 = 280$ $\Rightarrow t = 12 \pm \sqrt{280}$ <p>Colony eliminated when <math>t = 12 + \sqrt{280} = 28.733\dots</math></p> <p>Time taken = <math>28.733\dots - 12 = 16.7</math> hours (3 sf)</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Complete method to find the values of <math>t</math> for which <math>N = 0</math> May only find the larger time (which is fine)</p> <p>Complete method to find the time taken for the antibiotic to eliminate the colony (eg their time – 12) Correct time <u>with units</u> (awrt 16.7 hours)</p>
<p><b>12 (e)</b></p>	<ul style="list-style-type: none"> <li>the bacteria are unlikely to react the antibiotic instantly</li> <li>there may be a time delay between between adminstrations and first signs of die out</li> </ul>	<p>B1</p> <p>[1]</p>	<p>Correct reason why the assumption may not be correct</p>
<p><b>13 (a)</b></p>	$\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$ $\equiv \left(\frac{1 - \sin x}{\cos x}\right)^2$ $\equiv \frac{(1 - \sin x)^2}{\cos^2 x}$ $\equiv \frac{(1 - \sin x)^2}{1 - \sin^2 x}$ $\equiv \frac{(1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)}$ $\equiv \frac{1 - \sin x}{1 + \sin x}$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Replaces tanx with sinx/cosx</p> <p>Forms a common denominator and distributes the power of 2</p> <p><b>OR</b> expands to obtain <math>\frac{1}{\cos^2 x} - \frac{2 \sin x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}</math> oe</p> <p>Use of <math>\cos^2 x \equiv 1 - \sin^2 x</math> on denominator</p> <p>Complete and convincing proof with no errors seen</p>

<p><b>13 (b)</b></p>	$\frac{1 - \sin 2x}{1 + \sin 2x} = \frac{1}{5} \Rightarrow \sin 2x = \dots$ $\Rightarrow \sin 2x = \frac{2}{3}$ $2x = \sin^{-1}\left(\frac{2}{3}\right) = 41.81031$ <p>Other values in range are</p> $180 - 41.810\dots = 138.1896$ $-180 - 41.810\dots = -221.8103$ $-360 + 41.810\dots = -318.1896$ <p>So solutions are</p> $x = -159.09, -110.91, 20.91 \text{ and } 69.09 \text{ (2 dp)}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1 A1 cao</p> <p><b>[5]</b></p>	<p>Uses part (a) to simplify LHS and attempts to re-arrange for <math>\sin 2x</math> M0 if they use <math>x</math>, but allow recovery of this in later workings</p> <p>Obtains correct principal value of <math>2x</math></p> <p>Correct method to find at least one other value of <math>2x</math> in range</p> <p>Obtains two correct values of <math>x</math> (correct to at least 1 dp) Obtains all four values of <math>x</math> given to two decimal places Ignore any solutions given that are outside of the range A0 for any additional solutions in range</p> <p>Working in radians can score the M marks but not the A marks</p>
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<p><b>14 (a)</b></p>	$\frac{dy}{dx} = -2x - 3$ <p>At <math>P</math>, <math>\frac{dy}{dx} = -2\left(-\frac{1}{2}\right) - 3 = -2</math></p> <p>So gradient of normal at <math>P</math> is <math>\frac{1}{2}</math></p> <p>Equation of normal at <math>P</math> is then</p> $y - \frac{5}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right)$ $\Rightarrow y = \frac{1}{2}x + \frac{3}{2}$ $3\ln^2 x - \frac{3}{2}\ln x + \frac{1}{2}x = \frac{1}{2}x + \frac{3}{2}$ $\Rightarrow 3\ln^2 x - \frac{3}{2}\ln x - \frac{3}{2} = 0$ $\Rightarrow 2\ln^2 x - x - 1 = 0$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1 oe</p> <p>A1</p> <p>[5]</p>	<p>Obtains correct <math>dy/dx</math></p> <p>Substitutes <math>x = -1/2</math> to find gradient at <math>P</math> (may make a slip)</p> <p>Complete method to find the equation of the normal Their gradient of the normal must not be their “-2”</p> <p>Correct equation of the normal in any form</p> <p>Shows the result convincingly with no errors seen</p>
<p><b>14 (b)</b></p>	$(2\ln x + 1)(\ln x - 1) = 0$ $\Rightarrow \ln x = -\frac{1}{2}, \ln x = 1$ $\Rightarrow x = e^{-\frac{1}{2}}, x = e$ <p>Hence</p> $Q = \left(e^{-\frac{1}{2}}, \frac{3}{2} + \frac{1}{2}e^{-\frac{1}{2}}\right)$ $R = \left(e, \frac{3}{2} + \frac{1}{2}e\right)$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>Complete method to solve the quadratic for <math>x</math></p> <p>Correct values of <math>\ln(x)</math> Implied by correct values of <math>x</math> following sufficient working</p> <p>Correct exact coordinates of <math>Q</math></p> <p>Correct exact coordinates of <math>R</math></p>

<b>15 (a)</b>	$\overline{AB} = \begin{pmatrix} 4 \\ -1-p \end{pmatrix}$ $\overline{BC} = \begin{pmatrix} -6 \\ -8 \end{pmatrix}$	B1 B1 <b>[2]</b>	<u>Accept use of <b>i-j</b> notation instead of column vectors</u> Correct expression for $\overline{AB}$ Correct expression for $\overline{BC}$	
<b>15 (b)</b>	$2\sqrt{4^2 + (-1-p)^2} + 2\sqrt{6^2 + 8^2} = 30$ $\Rightarrow \sqrt{16 + 1 + 2p + p^2} + 10 = 15$ $\Rightarrow \sqrt{17 + 2p + p^2} = 5$ $\Rightarrow p^2 + 2p + 17 = 25$ $\Rightarrow p^2 + 2p - 8 = 0$ $\Rightarrow (p+4)(p-2) = 0$ <p>But since <math>p &gt; 0, p = 2</math></p>	M1 M1  M1  A1 <b>[4]</b>	Correct magnitude of <u>their</u> $\overline{AB}$ or their $\overline{BC}$ seen Forms an equation in terms of $p$ using the given information and <u>their</u> magnitudes  Employs a complete method to solve the resulting equation for $p$ Must be working from an equation in a comparable form  Obtains correct value of $p$ . Final answer, no errors	
<b>15 (c)</b>	$\overline{OD} = \overline{OA} + \overline{AD}$ $= \overline{OA} + \overline{BC}$ $= \begin{pmatrix} 3 \\ "2" \end{pmatrix} + \begin{pmatrix} -6 \\ -8 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ -6 \end{pmatrix}$	<b>OR:</b> $\overline{OD} = \overline{OC} + \overline{CD}$ $= \overline{OC} + \overline{BA}$ $= \begin{pmatrix} 1 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 - "2" \end{pmatrix}$ $= \begin{pmatrix} -3 \\ -6 \end{pmatrix}$	    M1  A1  <b>[2]</b>	<u>Accept use of <b>i-j</b> notation instead of column vectors</u>  Correct method to find the position vector of $D$ using <u>their</u> magnitudes and their $p$  Correct position vector of $D$

<p><b>16</b></p> <p>Equation of <math>l_1</math> is <math>y = 3x - 4</math>, so gradient of <math>l_1</math> is 3</p> <p><math>\Rightarrow</math> Gradient of <math>l_2</math> is <math>-\frac{1}{3}</math></p> <p>So equation of <math>l_2</math> is <math>y - 1 = -\frac{1}{3}(x - 3)</math> (<math>\Rightarrow y = -\frac{1}{3}x + 2</math>)</p> <p>Coordinates of <math>A = (0, 2)</math> Coordinates of <math>C = (4/3, 0)</math></p> <p>For <math>B</math>,</p> $3x - 4 = -\frac{1}{3}x + 2$ $\Rightarrow \frac{10}{3}x = 6$ $\Rightarrow x = \frac{9}{5}$ <p>Thus <math>y = \frac{7}{5}</math>, so coordinates of <math>B = \left(\frac{9}{5}, \frac{7}{5}\right)</math></p> $\text{Area of } OABC = \left(\frac{2 + 7/5}{2}\right)\left(\frac{9}{5}\right) - \frac{1}{2}\left(\frac{7}{5}\right)\left(\frac{9}{5} - \frac{4}{3}\right)$ $= \frac{41}{15}$		<p>M1</p> <p>M1</p> <p>A1 oe</p> <p>B1 A1ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[9]</p>	<p>This is an overall problem-solving mark to find one relevant area. For this, must see</p> <ul style="list-style-type: none"> <li>• correct method to find the eq of <math>l_2</math></li> <li>• attempt at the coordinates of <math>B</math> and <math>C</math> <b>and</b> coordinates of <math>A</math> correct</li> <li>• correct method used to find one relevant area (see table)</li> </ul> <p>Method to find gradient of <math>l_2</math> using negative reciprocal gradient</p> <p>Correct equation of <math>l_2</math> in any form</p> <p>Correct coordinates of <math>A</math> seen at any stage (may be on diagram) Correct coordinates of <math>C</math> seen at any stage (may be on diagram)</p> <p>Complete method to find the coordinates of <math>B</math> eg eliminating <math>y</math>, solving for <math>x</math> and substituting to find <math>y</math></p> <p>Correct coordinates of <math>B</math></p> <p>Complete method to find the area (see table and diagram overleaf) {Scheme uses trapezium <math>OABD</math> – triangle <math>BCD</math>}</p> <p>Correct area</p>
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**Turn over for the table**

## List of relevant areas for Q16 – referenced diagram on the next page

This table not exhaustive, but contains the areas anticipated to be most frequently appearing

Some shapes are grouped into boxes – this to show that these shapes are likely to be used together to give the required area (2nd M1)

Shape	Method(s) to find area	Shape	Method(s) to find area
<i>OABD</i> (trapezium)	$\frac{1}{2}(OA + BD) \times OD = \frac{1}{2}\left(2 + \frac{7}{5}\right) \times \frac{9}{5} = \frac{153}{50}$	<i>OAE</i> (triangle)	$\frac{1}{2}(OA)(OE) = \frac{1}{2}(2)(6) = 6$ <b>OR</b> $\int_0^6 \left(-\frac{1}{3}x + 2\right) dx$
<i>BCD</i> (triangle)	$\frac{1}{2}(BD)(CD) = \frac{1}{2}\left(\frac{7}{5}\right)\left(\frac{9}{5} - \frac{4}{3}\right) = \frac{49}{150}$ <b>OR</b> $\int_{\frac{4}{3}}^{\frac{9}{5}} (3x - 4) dx$	<i>BCE</i> (triangle)	$\frac{1}{2}(CE)(BD) = \frac{1}{2}\left(6 - \frac{4}{3}\right)\left(\frac{7}{5}\right) = \frac{49}{15}$ <b>OR</b> $\frac{1}{2}(CD)(BD) + \frac{1}{2}(BD)(DE)$ $= \frac{1}{2}\left(\frac{7}{5}\right)\left(\frac{9}{5} - \frac{4}{3}\right) + \frac{1}{2}\left(\frac{7}{5}\right)\left(6 - \frac{9}{5}\right)$ <b>OR</b> $\int_{\frac{4}{3}}^{\frac{9}{5}} (3x - 4) dx + \int_{\frac{9}{5}}^6 \left(-\frac{1}{3}x + 2\right) dx$
<i>ABF</i> (triangle)	$\frac{1}{2}(AF)(BG) = \frac{1}{2}(6)\left(\frac{9}{5}\right) = \frac{27}{5}$ <b>OR</b> $\frac{1}{2}(GF)(BG) + \frac{1}{2}(AG)(BG)$ $= \frac{1}{2}\left(\frac{7}{5} + 4\right)\left(\frac{9}{5}\right) + \frac{1}{2}\left(2 - \frac{7}{5}\right)\left(\frac{9}{5}\right)$	<i>OCF</i> (triangle)	$\frac{1}{2}(OC)(OF) = \frac{1}{2}\left(\frac{4}{3}\right)(4) = \frac{8}{3}$ <b>OR</b> $-\int_0^{\frac{4}{3}} (3x - 4) dx$

For the problem-solving mark, we need to see a “correct method used to find a relevant area”. For this M1, we need to see them use their values to find one of the relevant shapes in this table: either *OABD*, *BCD*, *OAE*, *BCE*, *ABF* or *OCF*. Values imply the shape they are working with.

The shapes in the table may be further sub-divided, but the areas of these smaller shapes are not sufficient for the M1.

For those using an integral, they must attempt the integral ( $x^n \rightarrow x^{n\pm 1}$ ) and substitute the limits in the correct order for this M1



