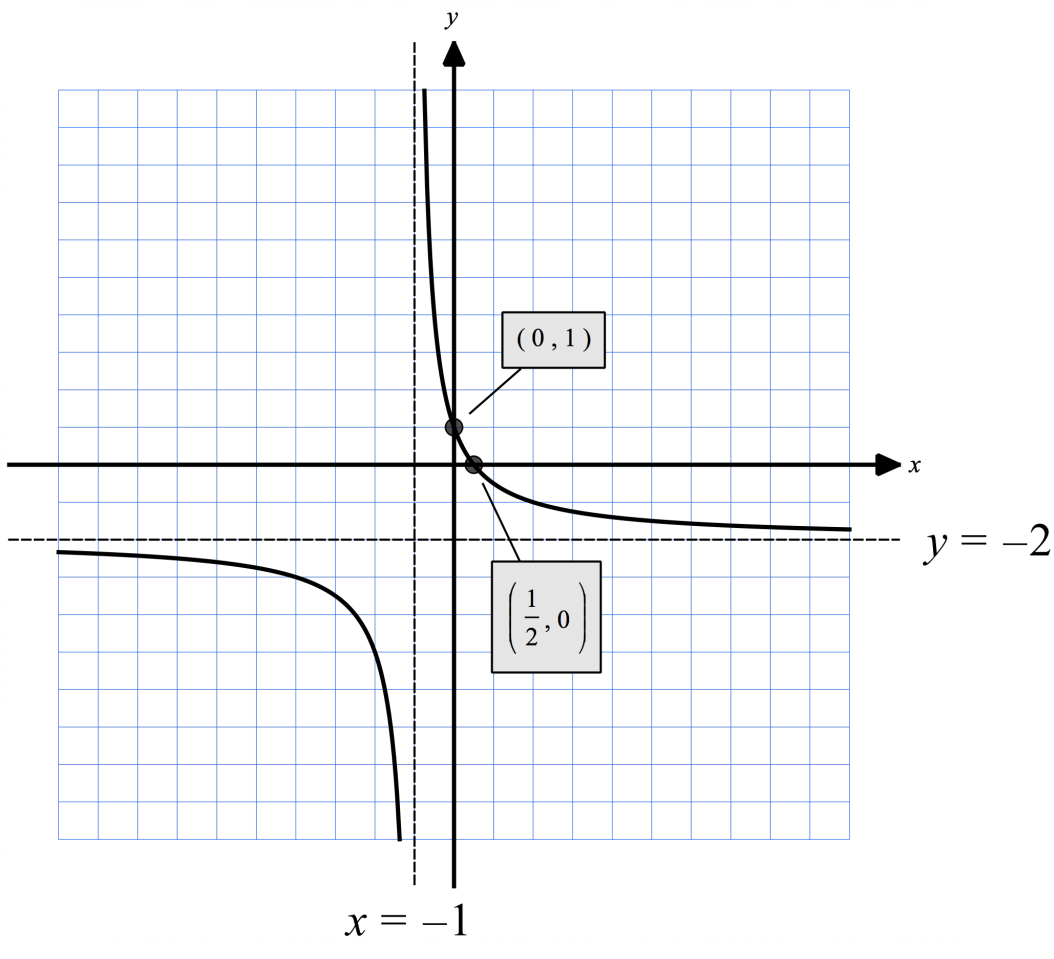
CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 9**

Model Solution No: 1



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Question Sheet: **Sheet 9**

Model Solution No: 2

(a) Note that $f'(x) = x + 7 - \left(x^{-\frac{1}{2}} - 2\right) = x - x^{-\frac{1}{2}} + 9$

Hence

$$\begin{aligned} y = f(x) &= \int f'(x) dx \\ &= \int \left(x - x^{-\frac{1}{2}} + 9\right) \, \mathrm{d}x \\ &= \frac{1}{2}x^2 - 2x^{\frac{1}{2}} + 9x + C \end{aligned}$$

Now we find the constant using the fact that when $x = 4, y = 20$:

$$20 = \frac{1}{2}(4)^2 - 2(4)^{\frac{1}{2}} + (4) + C \Rightarrow C = -20$$

Thus $y = \frac{1}{2}x^2 - 2x^{\frac{1}{2}} + 9x - 20$

Answer: $y = \frac{1}{2}x^2 - 2x^{\frac{1}{2}} + 9x - 20$

(b) The gradient of the tangent is $f'(1) = 1 + 7 - \frac{\sqrt{1}+2(1)}{1} = 9$

Furthermore when $x = 1$, $y = \frac{1}{2}(1)^2 - 2(1)^{\frac{1}{2}} + 9(1) - 20 = -\frac{25}{2}$

So the equation of the tangent is

$$y + \frac{25}{2} = 9(x - 1)$$

Then we know that Q is where C meets the y axis, so if you substitute $x = 0$ into the equation of the tangent and re-arrange for y, you will find the coordinates of $Q = \left(0, -\frac{43}{2}\right)$

Hence, the distance PQ is

$$\sqrt{(0-1)^2 + \left(-\frac{43}{2} - -\frac{25}{2}\right)^2} = \sqrt{82}$$

using distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (or just Pythagoras).

Answer: $\sqrt{82}$

Question Sheet: **Sheet 9**

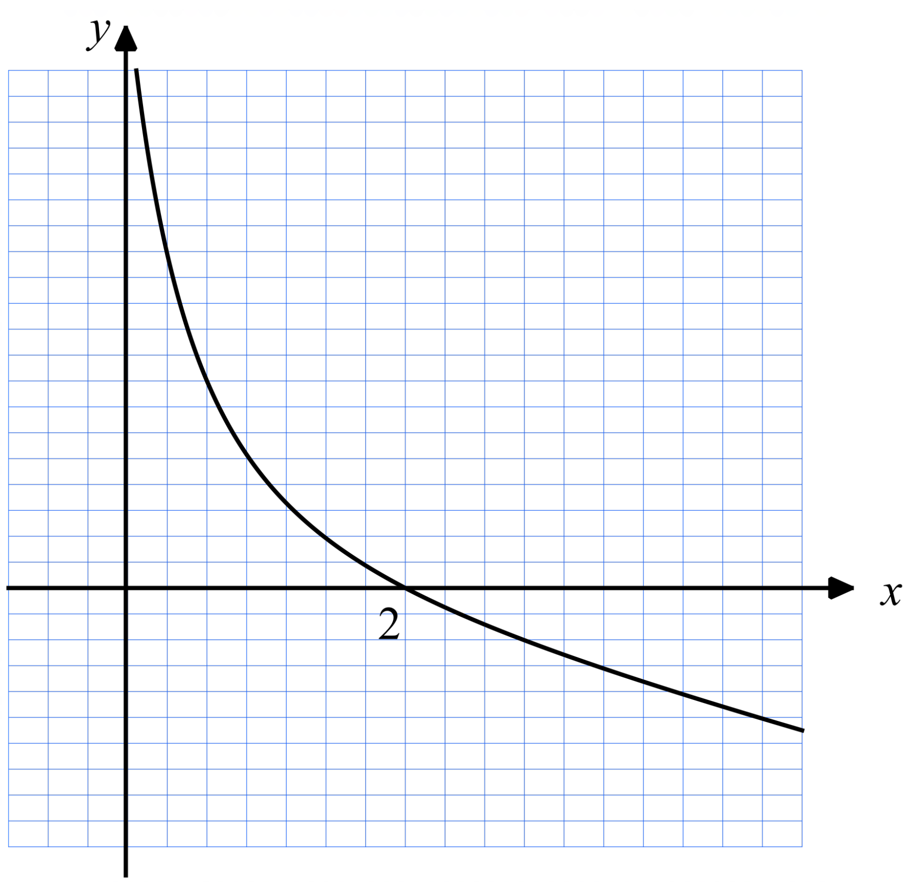
Model Solution No: 3

(a) **Answer:** $x = \frac{1}{2}, \quad x = \frac{3}{2}$

(b) **Answer:** $x = 3$

(c) **Answer:** $k > -8$

(d) **Answer:**



Question Sheet: **Sheet 9**

Model Solution No: 4

First we find the equation of the normal:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{10}{x^3} + \frac{12}{x^4}$$
$$\Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1} = -\frac{10}{1^3} + \frac{12}{1^4} = 2$$

Hence the gradient of the normal is $-\frac{1}{2}$

Also note that at $x = 1$, the y value is 1. Thus the equation of the normal is:

$$y - 1 = -\frac{1}{2}(x - 1)$$

We need the point where the normal crosses the y axis, which from the equation of the tangent with $x = 0$ is $(0, \frac{3}{2})$

The curve crosses the x axis when $0 = 5x - 4 \Rightarrow x = \frac{4}{5}$. Hence the area between the curve from $x = \frac{4}{5}$ to $x = 1$ is:

$$\int_{\frac{4}{5}}^{1} \left(5x^{-2} - 4x^{-3}\right)\, \mathrm{d}x = \left[-5x^{-1} + 2x^{-2}\right]_{\frac{4}{5}}^{1}$$
$$= -5(1)^{-1} + 2(1)^{-2} - (-5(4/5)^{-1} + 2(4/5)^{-2})$$
$$= \frac{1}{8}$$

Now to find the area of R, we just find the area of the quadrilateral (created by the x axis, y-axis, line l and by dropping a normal line from P to x axis).

This quadrilateral has parallel sides 1 and $\frac{3}{2}$ and height 1, so its area is $\frac{1}{2}(1 + \frac{3}{2})(1) = \frac{5}{4}$

Hence the area of R is $\frac{5}{4} - \frac{1}{8} = \frac{9}{8}$

Answer: $\frac{9}{8}$

Question Sheet: **Sheet 9**

Model Solution No: 5

(a) **Answer:** 4 m

(b) You need to expand the brackets, differentiate and then substitute $t = 60$. Details omitted due to time constraints.

Answer: 0.02432 m/s (you need the positive value of your answer, because the question asks for the rate of *decrease*)

(c) $H = 0 \Rightarrow (2 - 0.008t)^2 = 0$, which is solved when $2 - 0.008t = 0$. This gives $t = 250$ s

In minutes, the time is then $250/60 = 4.1666...$ m

Answer: 4.17 minutes

(d) **Answer:** Model valid for $(0 \leq)$ $t \leq 4.17$

Question Sheet: **Sheet 9**

Model Solution No: 6

(a)

$$\begin{aligned} v &= \int a dt \\ &= at + c \end{aligned}$$

Now at $t = 0$, $v = u$, so $u = a(0) + c \Rightarrow c = u$. So at any time t, we have that $v = u + at$ as required.

(b)

$$\begin{aligned} s &= \int v dt \\ &= \int (u + at) dt \\ &= ut + \frac{1}{2}at^2 + d \end{aligned}$$

Now at $t = 0$, $s = 0$, so $0 = u(0) + \frac{1}{2}a(0)^2 + d \Rightarrow d = 0$. So at any time t, we have that $s = ut + \frac{1}{2}at^2$ as required.

(c) Our goal is to eliminate t from the equation. So we take $v = u + at$, write it as $t = \frac{v-u}{a}$ and wherever we see t in the second equation, we replace it with that:

$$\begin{aligned} s = ut + \frac{1}{2}at^2 &\Rightarrow s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 \\ &\Rightarrow 2s = \frac{2uv - 2u^2}{a} + \frac{v^2 + u^2 - 2uv}{a} \\ &\Rightarrow 2as = 2uv - 2u^2 + v^2 + u^2 - 2uv \\ &\Rightarrow 2as = v^2 - u^2 \end{aligned}$$

as required.

Question Sheet: **Sheet 9**

Model Solution No: 7

(a) (i) **Answer:** temperatures quite high so suggest Beijing or Jacksonville

(a) (ii) **Answer:** e.g. all UK locations have roughly similar temperature/pressure patterns

(b) (i) **Answer:** e.g. for the squares/overseas location, as temperature increases, pressure decreases OR the data indicated by squares has a negative correlation

(b) (ii) **Answer:** e.g. no correlation/weak positive correlation between data marked by circles

(c) **Answer:** e.g. use a larger sample size / use more locations / consider overseas and UK locations separately

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