CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 8**

Model Solution No: 1

Note that $y = 3x^3 - x^4$. First we find the gradient of the tangent:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 9x^2 - 4x^3$$
$$\Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1} = 9(1)^2 - 4(1)^3 = 5$$

Hence the gradient of the normal is $-\frac{1}{5}$. Now when $x = 1$, we have $y = 1^3(3-1) = 2$. Hence the equation is

$$y - 2 = -\frac{1}{5}(x - 1)$$

and re-arranging gives the result

**Answer:** $x + 5y - 11 = 0$ (or any non-zero, integer multiple of this)

Question Sheet: **Sheet 8**

Model Solution No: 2

(a) **Solution:** $f(3) = 3^3 - 2(3)^2 - 5(3) + 6 = 27 - 18 - 15 + 6 = 0$. Hence by the factor theorem, since $f(3) = 0$, $(x - 3)$ is a factor of $f(x)$, as required.

(b) Use long division or inspection to find the other quadratic factor. It will turn out to be $(x^2 + x - 2)$.

Hence we have $f(x) = (x - 3)(x^2 + x - 2) = (x - 3)(x + 2)(x - 1)$

**Answer:** $f(x) = (x - 3)(x + 2)(x - 1)$

(c) This equation is just $f(4^x) = 0$, so it factorises to

$$(4^x - 3)(4^x + 2)(4^x - 1) = 0$$

This gives either $4^x = 3$, $4^x = -2$ or $4^x = 1$.

Now $4^x = -2$ is not valid, so we are left with $4^x = 3$ or $4^x = 1$

If $4^x = 3$, then taking logs gives $x \log 4 = \log 3 \Rightarrow x = \frac{\log 3}{\log 4} = 0.7924...$

Alternatively, we have $4^x = 1$, which is $x = 0$ by inspection (or you can use logs).

**Answer:** $x = 0$ or $x = 0.792$

Question Sheet: **Sheet 8**

Model Solution No: 3

(a) **Answer:** Using the binomial expansion, you will find $256 + 1024kx + 1792k^2x^2 + 1792k^3x^3 + \cdots$.

(b) Coefficient of $x^3$ is 5 times coefficient of $x^2$, so

$$1792k^3 = 5(1792k^2) \Rightarrow k^3 - 5k = 0$$

which factorises to $k^2(k-5) = 0$. Hence since $k \neq 0$, we must have $k = 5$

**Answer:** $k = 5$

Question Sheet: **Sheet 8**

Model Solution No: 4

(a) **Solution:**

$$\begin{aligned}\left(\frac{1}{\cos x} - \tan x\right)^2 &\equiv \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 \\ &\equiv \left(\frac{1-\sin x}{\cos x}\right)^2 \\ &\equiv \frac{(1-\sin x)^2}{\cos^2 x} \\ &\equiv \frac{(1-\sin x)^2}{1-\sin^2 x} \\ &\equiv \frac{(1-\sin x)^2}{(1-\sin x)(1+\sin x)} \\ &\equiv \frac{1-\sin x}{1+\sin x}\end{aligned}$$

(b) Using the previous part, we have

$$\frac{1-\sin x}{1+\sin x} = \frac{1}{5}$$

which re-arranges to

$$5(1-\sin x) = 1(1+\sin x)$$

and thus gives $\sin x = \frac{2}{3}$

Now you can use your favourite method to solve trigonometric equations to obtain the answers by solving $\sin x = \frac{2}{3}$ on the given interval. (We don't show this as people have different methods and we would rather not create confusion at this time of year.)

**Answer:** $x = 41.8$ or $x = 138$ (to 3 sf)

Question Sheet: **Sheet 8**

Model Solution No: 5

Since $M$ decreases exponentially, we have

$$M = A\mathrm{e}^{kt}$$

where $t$ is time. The initial mass was 120 since $A = 120$. Four years later, the mass is 72.4, so

$$72.4 = 120\mathrm{e}^{kt}$$

You can then take logs on both sides to find that $k = -0.1263...$

Hence $M = 120\mathrm{e}^{-0.1263...t}$.

Then after 10 years, we have

$$M(10) = 120\mathrm{e}^{-0.1263...(10)} = 33.93...$$

**Answer:** 33.9 g (3 sf)

Question Sheet: **Sheet 8**

Model Solution No: 6

(a) Considering the whole system and resolving upwards, we have

$$T - (0.8 + 0.2)g = (0.8 + 0.2)(3)$$

Re-arranging then gets the value of $T$.

**Answer:** $T = 12.8$ N or $T = 13$ N

(b) Consider block $A$, then

$$R - 0.2g = 0.2(3) \Rightarrow R = 2.56\text{N}$$

**Answer:** 2.6 N (or 2.56)

(c) **Answer:** By Newton's 3rd Law, the answer is the same as part (b), so 2.6 N (or 2.56).

(d) **Answer:** We have used the fact that the lift is light by taking the mass of the lift to be 0.

Question Sheet: **Sheet 7**

Model Solution No: 7

(a) Probability that die lands on 4 is $\frac{1}{4}$

If it lands on a 4, the coin is filled 6 times. Probability coin shows 4 heads in 4 throws is ${}^{6}\mathrm{C}_4(0.2)^4(1-0.2)^2 = 0.01536$

So to find the probability of both events, we multiply the probabilities together to obtain the answer.

**Answer:** 0.00384 (or equivalent, e.g. $\frac{12}{3125}$)

(b) The events are:

Die shows 1 and obtain 1 head $= \frac{1}{4} \times {}^{3}\mathrm{C}_1(0.2)^1(1-0.2)^2 = \frac{12}{125}$

Die shows 2 and obtain 2 heads $= \frac{1}{4} \times {}^{4}\mathrm{C}_2(0.2)^2(1-0.2)^2 = \frac{24}{625}$

Die shows 3 and obtain 3 heads $= \frac{1}{4} \times {}^{5}\mathrm{C}_3(0.2)^3(1-0.2)^2 = \frac{8}{625}$

Die shows 4 and obtain 4 heads and we found this probability in (a).

Then we just sum the probabilities together for the final answer

**Answer:** $\frac{472}{3125}$ (or awrt 0.151)

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