CM

AS Level Maths Question Countdown

4 days until the 1st exam

Information

- Each of the ten sheets will contain five pure questions and two applied questions.

Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

Notes to self

Pure questions – standard

1 Solve the simultaneous equations $5x + y = 1, \quad x^2 + (y-2)^2 = 1.$

Extension: slightly open-ended, but can you interpret your solutions geometrically?

2 (a) (i) Prove that the product of two consecutive integers is even.

(ii) Deduce that $n^2 + n + 1$ is odd for all integers n.

Jessice claims that "if n is an integer, then $n^2 - 1$ is prime."

(b) Disprove Jessica's claim using a suitable counterexample.

Extension: prove part (a) using ***exhaustion***. More concretely, answer the following question:

Using proof by exhaustion, prove that

$$n^2 + n + 1$$

is odd for all integers n.

Pure questions – problems

3 A curve C is such that $\dfrac{d^2y}{dx^2} = 12x - 4$.

Given that C has a stationary point at $(2, -1)$,

(a) show in clear stages that

$$y = 2x^3 - 2x^2 - 16x + k$$

where k is a constant to be found.

(b) Determine the coordinates of the other stationary point on C **and** determine its nature.

4 Given that $p = \log_4 x$, find, in terms of p, the simplest form of

(a) $\log_4(64x)$

(b) $\log_4\left(\dfrac{x^3}{4}\right)$

(c) Hence, or otherwise, solve the equation

$$\log_4(64x) - \log_4\left(\frac{x^3}{4}\right) = 3$$

Pure questions – modelling

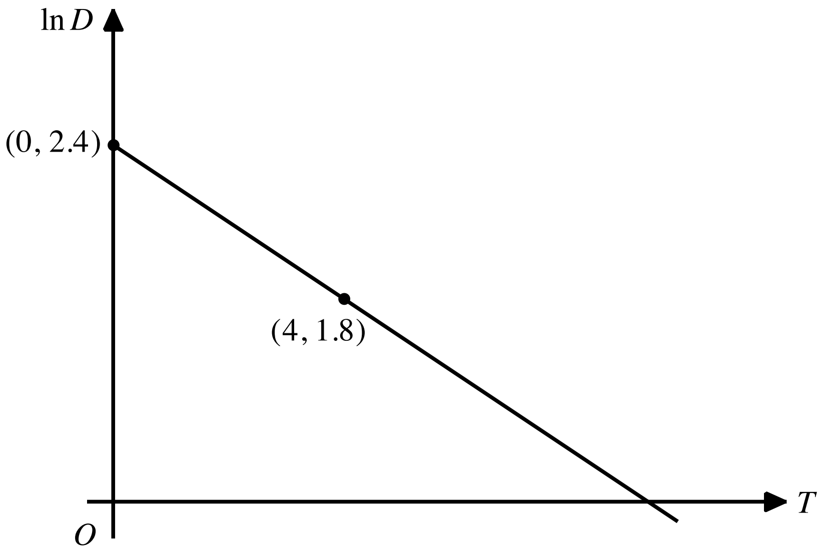
5 The amount of drug, D milligrams, in a patient's system at time T hours after receiving a dose is expected to satisfy the relationship

$$D = a\mathrm{e}^{bT}, \quad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\ln D = mT + c$$

giving m and c in terms of a and/or b.



The diagram above shows the graph of $\ln D$ against T for a particular patient A.

This graph passes through the points (0, 2.4) and (4, 1.8).

(b) Use the graph to find the values of a and b in the model for this case.

Five hours after patient A's initial dose, a second dose of 7 mg is administrated.

(c) Determine the total amount of drug in patient A's system immediately after the patient receives the second dose.

The same drug is administrated to a different patient B. Patient B receives a higher initial dose than patient A. It is believed that patient B breaks down the drug at a faster rate than patient A.

(d) Explain how the graph for $\ln D$ against T for patient B will be different to the graph for $\ln D$ against T for patient A.

Applied questions – mechanics

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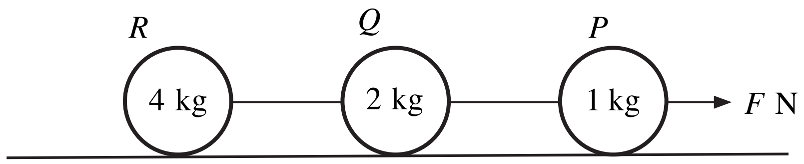

Particles P, Q and R have masses 1 kg, 2 kg and 4 kg respectively. The three particles lie on a rough horizontal surface. Light inextensible strings connect the particles P and Q and the particles Q and R. Initially, the particles lie at rest on the surface with the string taut.

It is modelled that each particle experiences a constant resistance to motion of the form kM N, where k is a constant and M is the mass of the particle.

A constant force F of magnitude 30 N is then applied to P in the direction QP, as shown in the diagram. The system accelerates at 3 m s^{-2}.

(a) Use the model to show that $k = \frac{9}{7}$.

(b) Calculate the tension in the string connecting P and Q.

(c) Calculate the tension in the string connecting Q and R.

(d) Explain how you have used the assumption that the string is inextensible.

After the particles have been moving for 5 seconds, both of the strings breaks. The particle P continues to move under the influence of F and the resistance to motion for each particle remains unchanged.

(e) Determine whether P comes to rest in its subsequent motion.

Applied questions – statistics

7 Jenny wants to create a probability model for the daily mean temperature, T °C, for a location in the large data set.

To do this, she looks at the 2015 data available for this location in the large data set. She creates four discrete categories for T and groups the data accordingly.

The table below shows her grouped frequency table.

Group	1	2	3	4
Daily mean temperature (°C)	0 – 10	10 – 20	20 – 30	30 – 40
Frequency	5	43	131	5

(a) Using your knowledge of the large data set, explain whether you think Jenny's location is in the UK or overseas.

Jenny then takes a random sample of 50 days from her data set.

(b) Define a suitable distribution to model the number of days that had a daily mean temperature greater than 20 °C.

(c) Using your distribution, find the probability that

(i) exactly 38 days had a daily mean temperature greater than 20 °C,

(ii) fewer than 10 days had a daily mean temperature less than 20 °C.

(d) Suggest **one** limitation of the large data set for Jenny's purposes.

Extension: using Jenny's groups, T can be viewed as a discrete random variable. Can you think of other examples?