CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 6**

Model Solution No: 1

Solution: The discriminant of this equation must equal zero.

Hence we must have

$$\Delta = (3 - m)^2 - 4(m)(1) = 0$$

Expanding and collecting like terms gives $m^2 - 10m + 9 = 0$. This factorises to $(m - 9)(m - 1) = 0$ and hence the possible values of m are $m = 1$ or $m = 9$.

Answer: $m = 1$ or $m = 9$

Question Sheet: **Sheet 6**

Model Solution No: 2

(a) **Solution:** The x coordinates of intersection points are the solutions of

$$x^3 - 4x = -\frac{4}{x}$$

Multiplying through by x we obtain

$$x^4 - 4x^2 = -4$$

and adding 4 to both sides, we thus have

$$x^4 - 4x^2 + 4 = 0$$

as required.

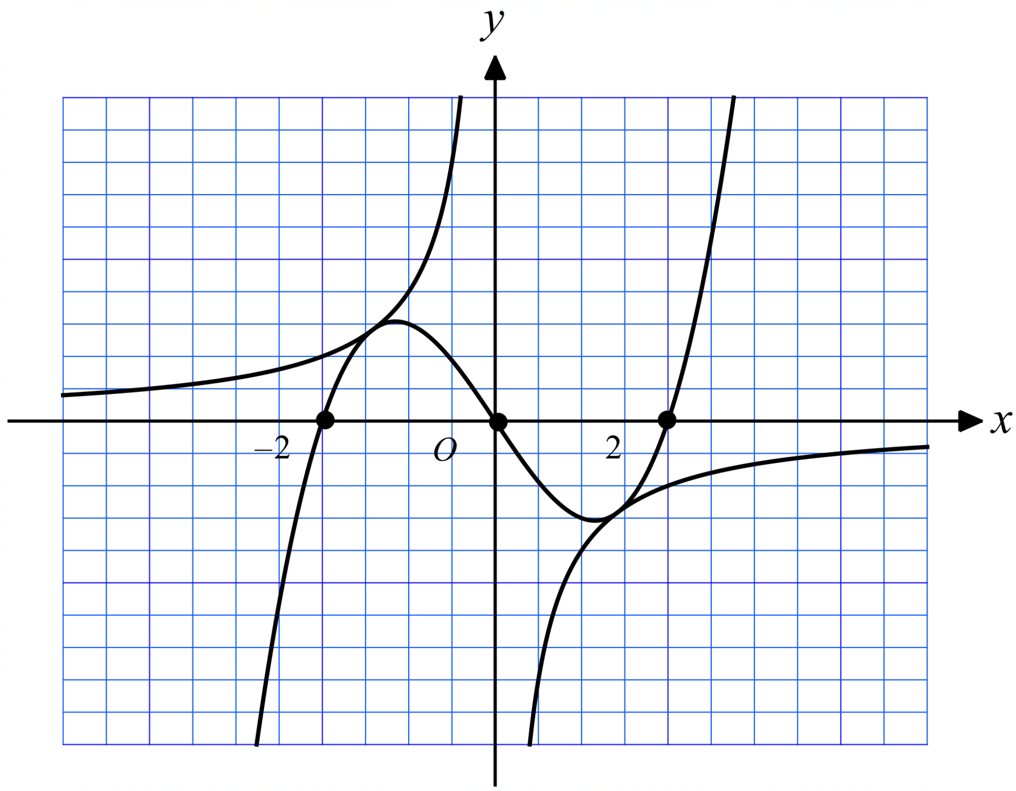
(b) This is a quadratic equation in disguise and factorises to $(x^2 - 2)^2 = 0$. [Use the substitution $y = x^2$ if it helps.]

Then we must have that $x^2 - 2 = 0$ or $x = \pm\sqrt{2}$.

We can substitute in to find the corresponding y coordinates and if you do this, you will obtain the required result.

Answer: $(\sqrt{2}, -2\sqrt{2})$ or $(-\sqrt{2}, 2\sqrt{2})$

(c) **Answer:** It is important here to have used the previous parts to ensure the graphs have two solutions only. You will lose marks for suggestions otherwise.



Question Sheet: **Sheet 6**

Model Solution No: 3

(a) Using the sine rule, we have

$$\frac{\sin\alpha}{10} = \frac{\sin 33}{6}$$

which gives $\sin\alpha = \frac{10\sin 33}{6}$

Thus

$$\alpha = \sin^{-1}\left(\frac{10\sin 33}{6}\right) = 65.19...$$

However, this is only the principal value. There is another (valid) solution to this equation, namely $180 - 65.19... = 114.8...$

Answer: $\alpha = 65.2$ or $\alpha = 115$ (3 sf)

(b) **Solution:** As above, we can use the sine rule here and we get to

$$\sin\beta = \frac{8\sin 64}{10}$$

If we inverse sine this time around, we get a principal value of $\beta = 45.974...$

We can again say that another solution to the equation is $180 - 45.974... = 134.026....$ *However*, this time around, while 134 is a mathematically valid solution, it is not valid geometrically.

This is because if β was 134, then the sum of it with the other angle in the triangle $(134 + 64)$ would equal 198 and this is larger than 180. Hence only $\beta = 46.0$ is valid

Question Sheet: **Sheet 6**

Model Solution No: 4

(a **Solution:** When $x = 0.5$, $f(0.5) = \frac{3}{0.5^3} - \frac{6}{0.5^2} = 24 - 24 = 0$. Hence $(1, 0)$ lies on C as required.

(b)

$$\begin{aligned}
\int_{\frac{1}{2}}^{2} (3x^{-3} - 6x^{-2})\,\mathrm{d}x &= \left[-\frac{3}{2}x^{-2} + 6x^{-1}\right]_{\frac{1}{2}}^{2} \\
&= \left(-\frac{3}{2}(2)^{-2} + 6(2)^{-1}\right) - \left(-\frac{3}{2}\left(\frac{1}{2}\right)^{-2} + 6\left(\frac{1}{2}\right)^{-1}\right) \\
&= \frac{21}{8} - 6 \\
&= -\frac{27}{8}
\end{aligned}$$

and thus the area of R is $\dfrac{27}{8}$

Answer: $\dfrac{27}{8}$

(b) (i) **Answer:** $-\dfrac{81}{8}$

(b) (ii) **Answer:** $-\dfrac{27}{8}$

Question Sheet: **Sheet 6**

Model Solution No: 5

(a) **Answer:** Height of the obstacle is 5 m and the length is 3 m.

(b) The quadratic equation has the form $H = ax^2 + bx$. The constant is 0 since it passes through the origin.

Since the ball passes through $(2, 5)$, we have $5 = a(2)^2 + b(2) \Rightarrow 5 = 4a + 2b$

Similarly, since the ball passes through $(5, 5)$, we have $5 = a(5)^2 + b(5) \Rightarrow 5 = 25a + 5b$

Solving these simultaneously for a and b, we have $a = -\frac{1}{2}$ and $b = \frac{7}{2}$

Answer: $H = -\dfrac{1}{2}x^2 + \dfrac{7}{2}x$

(c) Completing the square gives

$$\begin{aligned} H &= -\frac{1}{2}\left(x^2 - 7x\right) \\ &= -\frac{1}{2}\left[\left(x - \frac{7}{2}\right)^2 - \frac{49}{4}\right] \\ &= -\frac{1}{2}\left(x - \frac{7}{2}\right)^2 + \frac{49}{8} \end{aligned}$$

Hence the maximum height reached by the ball above the ground is $\frac{49}{8}$ m.

Answer: $H_{\text{max}} = \dfrac{49}{8}$ m.

(d) e.g. modelled ball as a particle **OR** ball cannot physically touch the corner of the obstacle / didn't take into account rebound/effect of ball from hitting corner **OR** model ignores air resistance/spin of ball (not exhaustive)

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Model Solution No: 6

(a) You need to add the vectors together and set each component equal to zero for overall zero force. Details omitted.

Answer: $a = -2$, $b = 1$

(b) Using Newton's Second Law in vector form ($\mathbf{F} = m\mathbf{a}$), the acceleration of the particle is

$$\mathbf{a} = \mathbf{i} - \frac{5}{2}\mathbf{j}$$

Thus the magnitude of the acceleration is $\sqrt{1^2 + \left(\frac{5}{2}\right)^2} = \frac{1}{2}\sqrt{29}$ m s^{-2}

The direction as a bearing is $90 + \tan^{-1}\left(\frac{5}{2}\right) = 158.19...$

Answer: magnitude $= \frac{1}{2}\sqrt{29}$ m s^{-2} direction = 158 given to nearest degree

Question Sheet: **Sheet 6**

Model Solution No: 7

(a) **Answer:** In order, the missing values for the table are: 60, 25, 30

(b) The total number of data values is 145, so the median is the 72.5th value. This lies in the class $10 - 20$ which has endpoints 10 and 20 and range 10.

Thus we use linear intepolation (note this may not be the same way you have done it):

$$\frac{Q_2 - 10}{20 - 10} = \frac{72.5 - 20}{80 - 20}$$

which if you re-arrange gives $Q_2 = 18.75$

Answer: 18.75

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