

# CM

## AS Level Maths Question Countdown

6 days until the 1<sup>st</sup> exam

### Information

- Each of the ten sheets will contain five pure questions and two applied questions.

#### Pure questions

- Two of the pure questions will be 'standard'.
- Two of the pure questions will be 'problems'.
- The last pure question will involve modelling.

#### Applied questions

- One of the questions will focus on statistics.
- One of the questions will focus on mechanics.
- On alternate days, the statistics question will look at the large data set. Note that these questions may be brief as opposed to full length exam questions.

### Notes to self

## Pure questions – standard

1 Without using a calculator, show that

$$\int_{1+\sqrt{2}}^4 \frac{1}{x^2} dx = \frac{1}{4}(a\sqrt{2} + b)$$

where  $a$  and  $b$  are constants to be found.

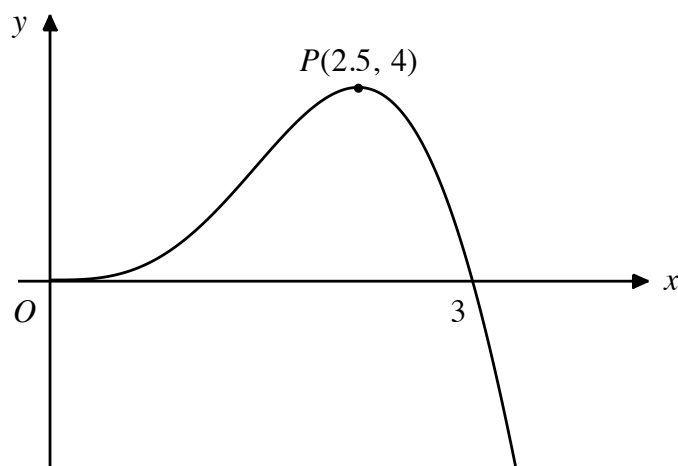
2 The curve  $C$  has the equation  $y = f(x)$ .

The point  $P(5, 2)$  lies on  $C$ .

Under the transformation  $y = f(x) + k$ , the point  $P$  is transformed to the point  $(5, -3)$ .

(a) State the value of  $k$ .

(b) Write down the coordinates of  $P$  under the transformation  $y = f(x - 2)$ .



The diagram above shows a sketch of another curve with equation  $y = g(x)$ .

The curve crosses the  $x$  axis at  $(0, 0)$  and  $(3, 0)$  and it has a maximum point at  $(2.5, 4)$ .

(c) Sketch the curve with equation  $y = g'(x)$ , showing the coordinates of any intersections with the  $x$  axis.

## Pure questions – problems

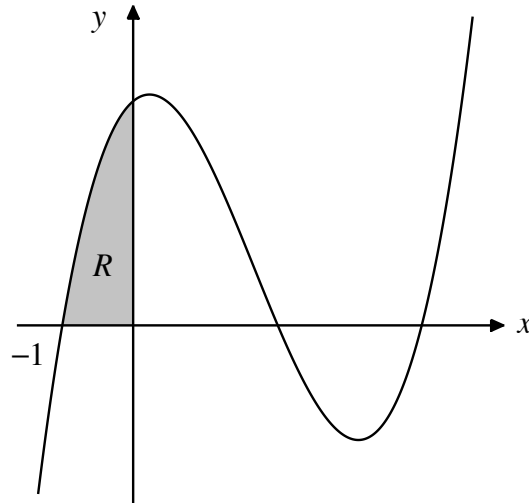
3 Points  $A$ ,  $B$  and  $C$  lie in a straight line such that the ratio  $AB : BC = 2 : 3$ .

Relative to a fixed origin  $O$ , points  $B$  and  $C$  have position vectors  $2\mathbf{i} + 3\mathbf{j}$  and  $\mathbf{i} - 6\mathbf{j}$  respectively.

(a) Showing your working, find the position vector of  $A$ .

(b) Hence find the distance of  $A$  from the origin  $O$ .

4



The diagram above shows a sketch of the curve  $C$  with equation  $y = f(x)$ , where

$$f(x) = x^3 + ax^2 + bx + 8, \quad \text{where } a \text{ and } b \text{ are constants.}$$

Given that  $C$  crosses the  $x$  axis at  $(-1, 0)$ .

- (a) (i) write down a factor of  $f(x)$ ,  
 (ii) and show that  $b - a = 7$ .

The region  $R$ , shown shaded in the diagram, is bounded by  $C$ , the  $x$  axis and the  $y$  axis.

Given that the area of  $R$  is  $\frac{61}{12}$ ,

- (b) show that  $3b - 2a = 16$ .  
 (c) (i) Deduce that  $a = -5$  and  $b = 2$ .  
 (ii) Hence express  $f(x)$  as a product of three linear factors.

### Pure questions - modelling

5 The size of a population,  $p$  thousands, is modelled to vary due to a disease according to

$$p = \frac{ae^{0.15t} - 1}{e^{0.3t} + 1}, \quad t \geq 0$$

The initial size of the population is 1500.

- (a) Show that  $a = 4$ .  
 (b) Use the model to show that the size of the population is never 2000.  
 (c) Show that  $p = \frac{4e^{-0.15t} - e^{-0.3t}}{1 + e^{-0.3t}}$ .  
 (d) Describe the long term impact of the disease on the population, justifying your answer.

## Applied questions – mechanics

- 6 A particle of mass 3 kg is moving on a smooth horizontal surface. At time  $t = 0$ , the particle passes through the point  $A$  and is moving at a constant speed of 15 m/s. After 8 s, the surface becomes rough and the particle is subject to a constant frictional force of magnitude 18 N. The particle subsequently comes to rest at the point  $B$  on the surface.
- Find the total time taken for the particle to come to rest.
  - Find the distance between the points  $A$  and  $B$ .

## Applied questions – statistics

- 7 Edward is investigating daily mean temperature,  $t$  °C, from the large data set.

He looks specifically at the data from 1987 Cambourne.

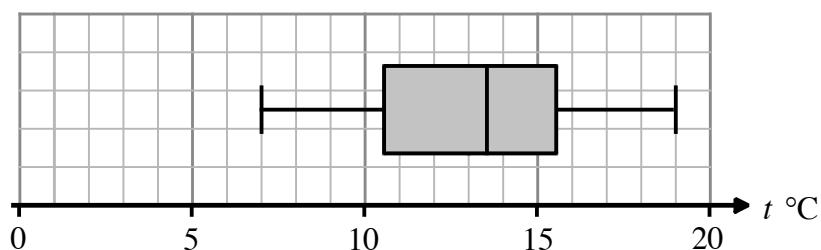
To obtain a sample of the data, he selects the first value using a random number from 1 and 4 and then selects every third value after that until he has a sample of size 12.

- (a) State the sampling technique used by Edward.

Clare states that,

“Edward’s sampling method may not always lead to a size 12 because there are gaps in the data set.”

- (b) Explain why Clare’s comment is not relevant for Edward’s investigation.



Edward now uses **all** of the data from 1987 Cambourne in the large data set to investigate daily mean temperature.

The data is summarised in the box plot above.

- (c) Determine the median and interquartile range of Edward’s results.

In 1987, the lowest recorded daily mean temperature in Cambourne was about  $-9$  °C. This is inconsistent with Edward’s box plot which has minimum temperature at  $7$  °C.

- (d) Using your knowledge of the large data set, suggest a reason for this inconsistency.