CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 5**

Model Solution No: 1

Solution: First note that $\frac{1}{x^2} = x^{-2}$. Then we have that:

$$\begin{aligned}
\int_{1+\sqrt{2}}^{4} \frac{1}{x^2}\,\mathrm{d}x &= \int_{1+\sqrt{2}}^{4} x^{-2}\,\mathrm{d}x \\
&= \left[-x^{-1}\right]_{1+\sqrt{2}}^{4} \\
&= -(4)^{-1} + (1+\sqrt{2})^{-1} \\
&= -\frac{1}{4} + \frac{1}{1+\sqrt{2}} \\
&= -\frac{1}{4} + \frac{1-\sqrt{2}}{-1} \qquad \text{(by rationalising)} \\
&= -\frac{1}{4} - 1 + \sqrt{2} \\
&= \sqrt{2} - \frac{5}{4} \\
&= \frac{1}{4}(4\sqrt{2} - 5)
\end{aligned}$$

as required with $a = 4$ and $b = -5$.

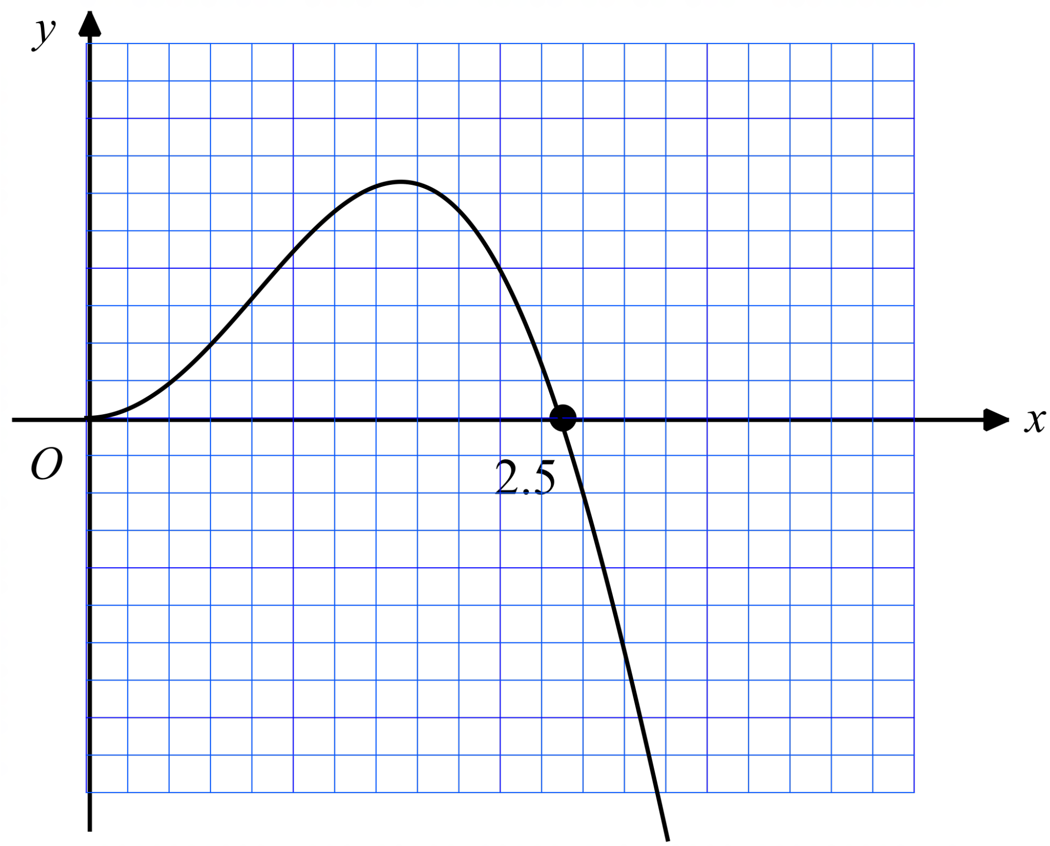
Question Sheet: **Sheet 5**

Model Solution No: 2

(a) **Answer:** $k = -5$

(b) **Answer:** $(7, 2)$

(c) **Answer:**



Question Sheet: **Sheet 5**

Model Solution No: 3

(a) $\overrightarrow{BC} = -\mathbf{i} - 9\mathbf{j}$

Then $\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BA}$. And since $\overrightarrow{BA} = -\overrightarrow{AB}$, we have that

$$\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB}$$

Now all that's left is to find what $\overrightarrow{AB}$ is.

We know that $AB : BC = 2 : 3$, so $\overrightarrow{AB} = \frac{2}{3}\overrightarrow{BC}$.

Hence, we have

$$\overrightarrow{OA} = (2\mathbf{i} + 3\mathbf{j}) - \frac{2}{3}(-\mathbf{i} - 9\mathbf{j})$$

Collecting like-terms then gives the result.

Answer: $\overrightarrow{OA} = \frac{8}{3}\mathbf{i} + 9\mathbf{j}$ (or equivalent)

(b) The distance is simply

$$\sqrt{\left(\frac{8}{3}\right)^2 + 9^2} = \frac{1}{3}\sqrt{739}$$

Answer: $\frac{1}{3}\sqrt{739}$ (or awrt 9.39)

Question Sheet: **Sheet 5**

Model Solution No: 4

(a) (i) **Answer:** $(x+1)$

(ii) **Solution:** Using the factor theorem (or the fact that $(-1,0)$ must satisfy the equation), we have that

$$f(-1)=0 \Rightarrow (-1)^3+a(-1)^2+b(-1)+8=0$$

which becomes $-1+a-b+8=0 \Rightarrow b-a=7$

(b) **Solution:** First let's find

$$\begin{aligned}\int_{-1}^{0}(x^3+ax^2+bx+8)\,\mathrm{d}x &= \left[\frac{x^4}{4}+\frac{a}{3}x^3+\frac{b}{2}x^2+8x\right]_{-1}^{0} \\ &= 0-\left(\frac{1}{4}-\frac{a}{3}+\frac{b}{2}-8\right) \\ &= \frac{a}{3}-\frac{b}{2}+\frac{31}{4}\end{aligned}$$

We are told that the area of R equals $\frac{61}{12}$, so we must have that

$$\frac{a}{3}-\frac{b}{2}+\frac{31}{4}=\frac{61}{12}$$

Re-arranging then gives the result. You can show this for yourself - if you get lost/stuck, try multiplying both sides by -6.

(c) (i) **Solution:** Solve the equations $b-a=7$ and $3b-2a=16$ simultaneously and you will get the result.

(c) (ii) Substituting in $a=-5$ and $b=2$, we have $f(x)=x^3-5x^2+2x+8$.

We know that one of the factors of this expression is $(x+1)$, so by inspection the other factor has to be (x^2-6x+8) (or you can work this out using long division). Thus

$$f(x)=(x+1)(x^2-6x+8)=(x+1)(x-2)(x-4)$$

Answer: $f(x)=(x+1)(x-2)(x-4)$

Question Sheet: **Sheet 5**

Model Solution No: 5

(a) **Solution:** When $t = 0$, $p = 1.5$ (remember p is in thousands!). Hence we have

$$\begin{aligned} 1.5 &= \frac{ae^0 - 1}{e^0 + 1} \\ \Rightarrow 1.5 &= \frac{a-1}{2} \\ \Rightarrow a - 1 &= 3 \\ \Rightarrow a &= 4 \end{aligned} \tag{1}$$

as required.

(b) **Solution:**

$$\begin{aligned} 2 &= \frac{4e^{0.15t} - 1}{e^{0.3t} + 1} \\ \Rightarrow 2e^{0.3t} + 2 &= 4e^{0.15t} - 1 \\ \Rightarrow 2e^{0.3t} - 4e^{0.15t} + 3 &= 0 \end{aligned} \tag{2}$$

This equation is equivalent to the quadratic $2x^2 - 4x + 3 = 0$ using the substitution $x = e^{0.15t}$. This has no real roots since the discriminant $\Delta = (-4)^2 - 4(2)(3) = -8$ is negative. So our equation can also have no real roots.

Hence the population can never reach 2000.

(c) **Solution:** Divide top and bottom of the model by $e^{0.3t}$ to get the result.

(d) **Answer:** As $t \to \infty$, we have $p \to 0$ (think about what is happening to each of the exponential terms as t gets large).

Hence the disease kills/destroys/eliminates the population

Question Sheet: **Sheet 5**

Model Solution No: 6

We split this up into two parts.

Smooth surface:

The particle moves on the smooth surface for 8 seconds at a constant speed 15 m/s. This means it travels a distance of 120 m during this period.

Rough surface:

First we need to find the deceleration of the particle when it moves on the rough surface. This is given by Newton's Second Law:

$$F = ma \Rightarrow 18 = 3(a)$$

so the particles decelerates at 6 m s^{-2}.

Now we can use SUVAT to find the distance the particle travels on the rough surface before coming to rest and the time this takes. We have:

$$s =? \quad u = 15\text{m/s} \quad v = 0 \quad a = -6\text{m/s}^2 \quad t =?$$

For s, we can use $v^2 = u^2 + 2as$ to obtain $s = \frac{75}{4}$.

We can use $v = u + at$ to find the time it takes for the particle to come to rest. The result is $t = \frac{5}{2}$ s.

(i) **Answer:** so total time taken to come to rest is $8 + \frac{5}{2} = \frac{21}{2}$

(ii) **Answer:** total distance travelled by particle is $120 + \frac{75}{4} = \frac{555}{4}$

Question Sheet: **Sheet 4**

Model Solution No: 7

(a) **Answer:** systematic sampling.

(b) **Answer:** e.g. there are no gaps for daily mean temperature in the data set/in Cambourne in 1987

(c) **Answer:** Median is 13.5. Interquartile range is 12

(d) **Answer:** e.g. LDS only contains data for March-October (so the minimum of -9 could have occurred outside this range)

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