CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 2**

Model Solution No: 1

(a) We have $4 - 5x \geq 2 - x$, which gives $4x \leq 2$. And so $x \leq \frac{1}{2}$

Answer: $x \leq \frac{1}{2}$

(b) We have to multiply both sides by x^2 because we don't know if x is positive or negative (but we do know that $x^2 \geq 0$ for all x).

Doing this gives

$$(5 - 3x)x > 2x^2$$

and then re-arranging we have $5x^2 - 5x < 0$ or $x(x - 1) < 0$. The critical values for the inequality are thus $x = 0$ and $x = 1$.

If you draw a graph or use your favourite method for solving quadratic inequalities, you can convince yourself that the solution is $0 < x < 1$.

Answer: $0 < x < 1$

[Alternatively for (b): you can consider cases. If $x > 0$, the inequality re-arranges to $5 - 3x > 2$, which gives $x < 1$. If $x < 0$, the inequality re-arranges to $5 - 3x < 2$ or $x > 1$, which is impossible (given $x < 0$). Hence solution is $0 < x < 1$.]

(c) We want to satisfy both inequalities. So we want the values of x that are in both these regions: $x \leq \frac{1}{2}$ AND $0 < x < 1$.

The values of x that are in both regions are $0 < x \leq \frac{1}{2}$. In set notation, the solution is written as: $\{x \in \mathbb{R} : 0 < x \leq \frac{1}{2}\}$.

Answer: $\{x \in \mathbb{R} : 0 < x \leq \frac{1}{2}\}$

Equivalent answers such as $\{x \in \mathbb{R} : x > 0\} \cap \{x \in \mathbb{R} : x \leq \frac{1}{2}\}$ are also acceptable. You are allowed to omit the '$\in \mathbb{R}$' also.

Question Sheet: **Sheet 2**

Model Solution No: 2

(a) We use the binomial expansion to get

$$(3-4x)^6 = 3^6 + {}^6\mathrm{C}_1(3)^5(-4x)^1 + {}^6\mathrm{C}_2(3)^4(-4x)^2 + \cdots$$

which simplifies down to $729 - 5832x + 19440x^2 + \cdots$

Answer: $729 - 5832x + 19440x^2$

(b) [NB: we begin with a fairly detailed explanation of the method, but this level of explanation will obviously not be required in a solution.]

One method is to just work out the expansion up until y^5 and then compare coefficients. However, this involves an unnecessary amount of work and if the power on the bracket was 99, and we were interested in the term in y^{76}, this approach is far from ideal.

A quicker way is to think about how we would obtain y^5 from the expansion. If we were to write it out, the expansion has 8 brackets:

$$(1+py)^8 = (1+py)(1+py)(1+py)(1+py)(1+py)(1+py)(1+py)(1+py)$$

To get a term in y^5, we want to choose the (py) term from exactly 5 of the brackets and from the other brackets, we need to choose 1.

Hence our term in y^5 will look like $A(py)^5(1)^3$, where A is the number of ways we can choose (py) five times from the eight brackets. By definition, this is ${}^8\mathrm{C}_5$.

The term in y^5 in our expansion is thus ${}^8\mathrm{C}_5(py)^5(1)^3 = 56p^5y^5$. We are told the coefficient should equal 13608, which gives

$$56p^5 = 13608$$

and hence $p = 3$.

Answer: $p = 3$

(c) This is similar to (b). Using $p = 3$, the term in y^6 is given by ${}^8\mathrm{C}_6(3y)^6(1)^2$, and so the coefficient of y^6 is 20412.

Answer: 20412

CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 2**

Model Solution No: 3

(a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = (7\mathbf{i} - \mathbf{j}) - (3\mathbf{i} + p\mathbf{j}) = 4\mathbf{i} + (-1-p)\mathbf{j}$

Similarly, $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = (\mathbf{i} - 9\mathbf{j}) - (7\mathbf{i} - \mathbf{j}) = -6\mathbf{i} - 8\mathbf{j}$

Answer: $\overrightarrow{AB} = 4\mathbf{i} + (-1-p)\mathbf{j}, \quad \overrightarrow{BC} = -6\mathbf{i} - 8\mathbf{j}$

(b) This is parallelogram, so the perimeter is $2|\overrightarrow{AB}| + 2|\overrightarrow{BC}|$, because opposite sides are equal in length.

Using part (a), $|\overrightarrow{AB}| = \sqrt{4^2 + (-1-p)^2}$ and $|\overrightarrow{BC}| = \sqrt{6^2 + 8^2} = 10$.

Putting everything together, we know that

$$\begin{aligned}
&2\sqrt{16 + (p+1)^2} + 2(10) = 30 \\
&\Rightarrow \sqrt{16 + (p+1)^2} = 5 \\
&\Rightarrow 16 + (p+1)^2 = 25 \\
&\Rightarrow (p+1)^2 = 9 \\
&\Rightarrow p + 1 = \pm 3 \\
&\Rightarrow p = -1 \pm 3
\end{aligned}$$

But since $p > 0$, $p = 2$

Answer: $p = 2$

(c) The position vector of D must be given by

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OC} + \overrightarrow{BA}$$

Hence $OD = (\mathbf{i} - 9\mathbf{j}) + (-4\mathbf{i} + 3\mathbf{j}) = -3\mathbf{i} - 6\mathbf{j}$

Answer: $-3\mathbf{i} - 6\mathbf{j}$

[Alternatively: you can do $OD = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{AB}$, which gives the same result.]

Question Sheet: **Sheet 2**

Model Solution No: 4

We have $y = 4x - x^{-\frac{1}{2}}$. Hence for the curve C,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + \frac{1}{2}x^{-\frac{3}{2}}$$

The normal to C at P is parallel to the line $16x + 65y - 10 = 0$. This line can be written in the more suggestive form $y = -\frac{16}{65}x + \frac{10}{65}$, which tells us that the line has gradient $-\frac{16}{65}$

The normal is parallel to this line, so also has gradient $-\frac{16}{65}$. Hence at P, the gradient of the *curve* must be $\frac{65}{16}$.

Hence the x coordinate at P satisfies

$$\begin{aligned} 4 + \frac{1}{2}x^{-\frac{3}{2}} &= \frac{65}{16} \\ \Rightarrow x^{-\frac{3}{2}} &= \frac{1}{8} \\ \Rightarrow x^{\frac{3}{2}} &= 8 \\ \Rightarrow x &= (\sqrt[3]{8})^2 = 4 \end{aligned}$$

So the x coordinate of P is 4. Then the y coordinate is found by substitution of $x = 4$ into the equation of C, which gives $y = 4(4) - (4)^{-\frac{1}{2}} = \frac{31}{2}$.

Answer: Coordinates of $P = \left(4,\ \frac{31}{2}\right)$

Question Sheet: **Sheet 2**

Model Solution No: 5

Since the rate of flow of liquid out of the tank is proportional to the amount of liquid in the tank, our model for V must be exponential, i.e. it must have the form

$$V = A\mathrm{e}^{kt}$$

We know the initial amount of water in the tank is $300\,\mathrm{cm}^3$, so $A = 300$. Then to find k, we use that at $t = 2$, $V = 80$, so

$$\begin{aligned} 80 &= 300\mathrm{e}^{2k} \\ \Rightarrow \mathrm{e}^{2k} &= \frac{8}{30} \\ \Rightarrow \ln(\mathrm{e}^{2k}) &= \ln\frac{8}{30} \\ \Rightarrow 2k &= \ln\frac{8}{30} \\ \Rightarrow k &= -0.6608... \end{aligned}$$

so our model is $V = 300\mathrm{e}^{-0.661t}$.

Answer: $V = 300\mathrm{e}^{-0.661t}$.

For Edexcel candidates, unless stated otherwise, always give values to three significant figures. (See front of the question paper)

Question Sheet: **Sheet 2**

Model Solution No: 6

We will use Newton's Second Law ($F = ma$) vertically and horizontally to form two equations which we can solve for a and b.

Resolving vertically, we have

$$2a - (b - 3) = 2(0) \Rightarrow 2a = b - 3$$

This is the expected answer of course (the particle doesn't move vertically so the forces should balance).

Similarly, resolving horizontally and to the right, we have

$$(a + b) - 5 = 2(2) \Rightarrow a + b = 9$$

Now we have two simultaneous equations and if you solve these, you will obtain the values for a and b.

Answer: $a = 2, b = 7$

Question Sheet: **Sheet 2**

Model Solution No: 7

Solution:

Let X be the number of heads obtained in 24 tosses of a coin. Then $X \sim \mathrm{B}(24, p)$.

The hypotheses are then: $\mathrm{H}_0 : p = 0.5$, $\mathrm{H}_1 : p > 0.5$.

We want to test the probability of getting 14 heads (or anything worse), so the probability of interest is

$$\mathbb{P}[X \geq 14 \mid X \sim \mathrm{B}(24, 0.5)] = 0.2706...$$

Then $0.2706... > 0.05$, hence there is insufficient evidence to reject the null hypothesis.

$\therefore$ There is no evidence to suggest the coin is biased towards heads.

[**Further insight:** Once we get to 0.2706, why does $0.2706 > 0.05$ mean we don't reject H_0? To answer this, you have to think about what probability we've working out. We calculated the probability that $X \geq 14$ given that the null hypothesis is true. We thus showed that if the coin is fair, there is about a 27% chance of seeing 14 heads or more.

This is bigger than the given significance level of 5%, so the probability of seeing 14 heads or more on a fair coin is not unlikely enough (relative to the given significance level) to be considered statistically significant.]

crashMATHS