CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 10**

Model Solution No: 1

(a) The gradient of l_1 is

$$m_{l_1} = \frac{6-3}{-2-4} = -\frac{1}{2}$$

so the gradient of $l_2 = 2$. Thus the equation of l_2 is

$$y - 5 = 2(x - 2)$$

Then re-arranging gives the result.

Answer: $2x - y + 1 = 0$ (or non-zero integer multiples)

(b) When $x = 0$, $y = 1$. So l_2 crosses the y axis a $(0, 1)$.

When $y = 0$, $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$, so l_2 crosses the x axis at $\left(-\frac{1}{2}, 0\right)$.

Hence the area of the triangle OAB is

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}\left(\frac{1}{2}\right)(1) = \frac{1}{4}$$

Answer: $\frac{1}{4}$

Question Sheet: **Sheet 10**

Model Solution No: 2

(a) Coordinates of intersection given by

$$3x^2 + 4x + 7 = 9 - x$$

which re-arranges to $3x^2 + 5x - 2 = 0$. This factorises to $(3x - 1)(x + 2) = 0$ and so the x coordinates of the intersection points are $x = \frac{1}{3}$ and $x = -2$.

You then substitute these back into either equation to find the y coordinates (details omitted). [NB: it's probably easier to substitute them back into the linear.]

Answer: Coordinates of intersection $(-2, 11)$ and $(\frac{1}{3}, \frac{26}{3})$

Easiest thing to do is to draw a sketch!!!!!

Once you have that, mark on your intersection points from part (a) and convince yourself of the answer.

Answer: $\{x \in \mathbb{R} : -2 < x < \frac{1}{3}\}$ (non-strict inequalities are acceptable also and other equivalent sets)

Question Sheet: **Sheet 10**

Model Solution No: 3

(a) $\frac{\mathrm{d}y}{\mathrm{d}x} = 3ax^2 + 10x + b$

We know that at $x = -3$ and $x = -\frac{1}{3}$, the derivative must be zero since these are stationary points. Hence we can form two equations:

From $x = -3$, we have: $0 = 3a(-3)^2 + 10(-3) + b \Rightarrow 27a + b = 30$

From $x = -\frac{1}{3}$, we have: $0 = 3a(-\frac{1}{3})^2 + 10(-\frac{1}{3}) + b \Rightarrow a + 3b = 10$

Solving these simultaneously will then give the values for a and b

Answer: $a = 1$, $b = 3$

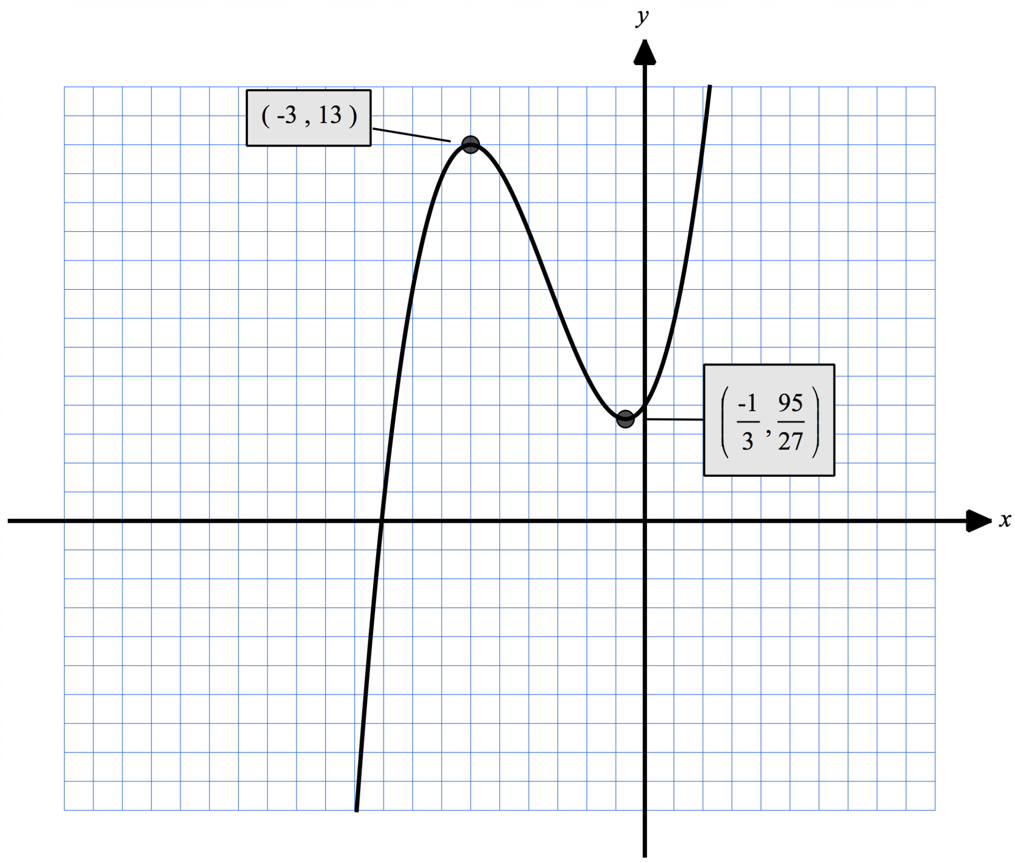
(b) **Solution:** $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x + 10$

At $x = -3$, the second derivative is $6(-3) + 10 = -8$. $-8 < 0$ and so $x = -3$ is a maximum point on the curve.

At $x = -\frac{1}{3}$, the second derivative is $6(-\frac{1}{3}) + 10 = 8$. $8 > 0$ and so $x = -\frac{1}{3}$ is a minimum point on the curve.

[NB: this is exactly what you expect from a cubic curve with positive leading coefficient.]

(c) **Answer:**


y
x
(-3 , 13)
$\left(\frac{-1}{3}, \frac{95}{27}\right)$

Question Sheet: **Sheet 10**

Model Solution No: 4

The equation of the circle is $(x-3)^2 + (y-1)^2 = 25$

The gradient of $3x + 4y + 20 = 0$ is $-\frac{3}{4}$, so the tangents must have equations of the form $y = -\frac{3}{4}x + k$

Let's find where this line meets the curve:

$$\begin{aligned}
&(x-3)^2 + \left(-\frac{3}{4}x + k - 1\right)^2 = 25 \\
&\Rightarrow x^2 - 6x + 9 + \frac{9}{16}x^2 - \frac{3}{2}x(k-1) + (k-1)^2 = 25 \\
&\Rightarrow \frac{25}{16}x^2 + \left(-\frac{9}{2} - \frac{3}{2}k\right)x + ((k-1)^2 - 16) = 0
\end{aligned}$$

This is a quadratic equation in x. For the lines to be tangents, we need only one solution, i.e. for the discriminant to be 0. Thus:

$$\Delta = b^2 - 4ac = \left(-\frac{9}{2} - \frac{3}{2}k\right)^2 - 4\left(\frac{25}{16}\right)((k-1)^2 - 16) = 0$$

If you expand the brackets and collect the like terms, you obtain the much more pleasant equation:

$$4k^2 - 26k - 114k = 0$$

which has solutions $k = -3$ and $k = \frac{19}{2}$.

Answer: $y = -\frac{3}{4}x - 3$ or $y = -\frac{3}{4}x + \frac{19}{2}$

Note 1: this is way above exam level expectations when it comes to the amount of algebraic manipulation needed. You could get a similar question with expressions that check out easier, e.g. the centre lies on the x axis.

Note 2: there is an easier way to do this problem, by ensuring the distance between the centre (point) and the line is 5, but you are not expected to be able to do this. However, any Further Mathematicians may like to try this alternative approach though (hint: use the 'vector' formula for distance between point and line and set it equal to 5)

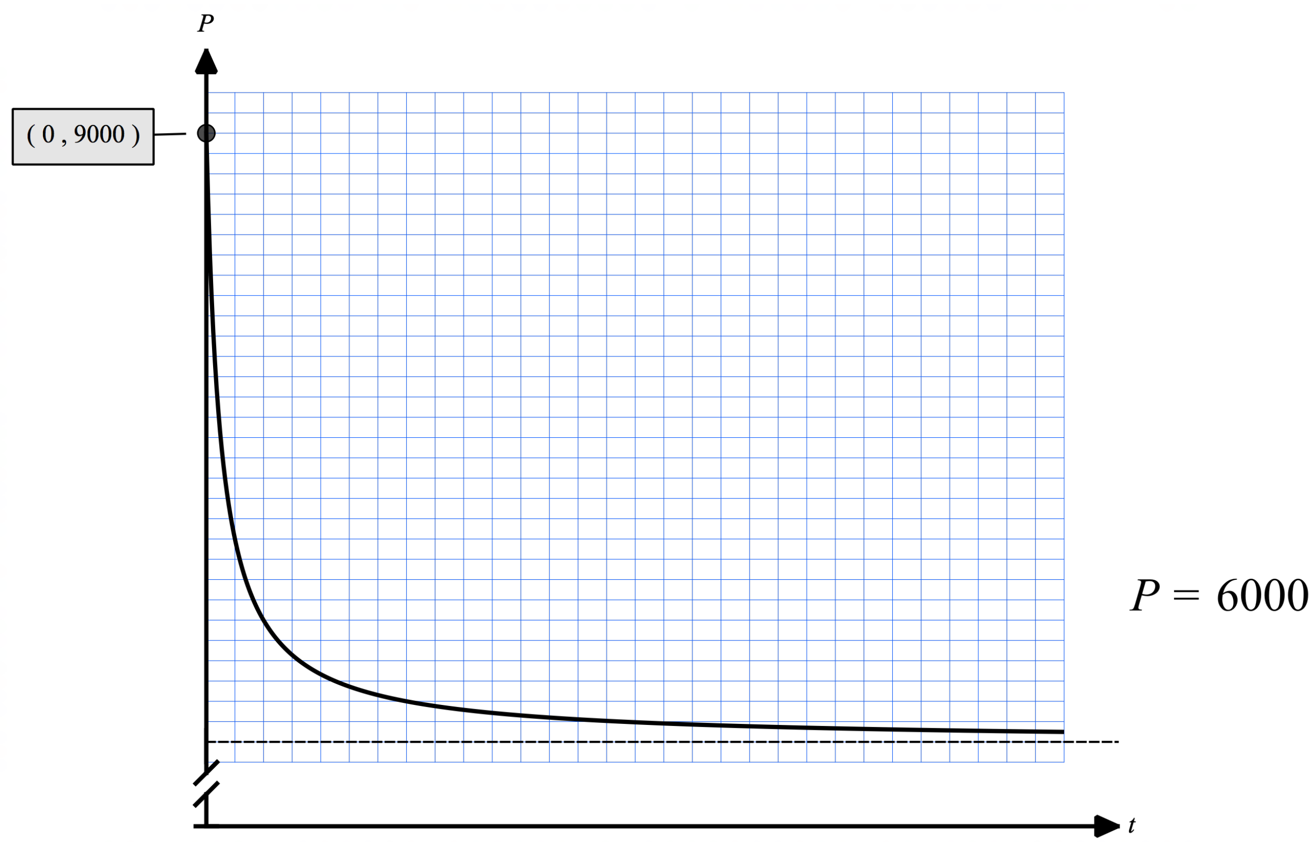
Question Sheet: **Sheet 10**

Model Solution No: 5

(a) **Answer:** 9000

(b) **Answer:** Substitute and re-arrange to obtain $T = 2.5$

(c) **Answer:**



(ignore the broken scale part of the sketch - it is the shape with asymptote to 6000 that is important)

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Question Sheet: **Sheet 10**

Model Solution No: 6

(a) The equations of motion for the two particles are:

$$R_P(\uparrow^+): \quad T - 2g = 2a$$

and

$$R_Q(\downarrow^+): \quad 5g - T = 5a$$

Adding these two equations together gives

$$3g = 7a \Rightarrow a = \frac{3}{7}g$$

So the acceleration of the two particles is $\boxed{\frac{3}{7}g \text{ m s}^{-2}}$

(b) The particle Q is 3 m above the ground and moves to the ground from rest with constant acceleration, so this is a 'SUVAT' equation problem. Using $s = ut + \frac{1}{2}at^2$, we have

$$3 = 0(t) + \frac{1}{2}\left(\frac{3}{7}g\right)t^2 \Rightarrow t = \sqrt{\frac{42}{3g}}$$

and so it takes Q approximately $\boxed{\text{1.2 seconds}}$ to reach the ground.

(c) It is important to try and think about what the system does and to break the problem down systematically. Once Q hits the ground, the string is no longer taut, so P will move under the influence of gravity until it reaches a maximum height and starts to fall down. To find the maximum height, we need to know the speed of P when Q hits the ground - this will be the same as the speed of Q when it hits the ground.

The speed of Q when it hits the ground is

$$v = \sqrt{2\left(\frac{3}{7}g\right)(3)} \approx 5.0199...$$

Therefore the maximum height P reaches above the ground can be solved using the 'SUVAT' equations with $s = ?$, $u = 5.0199...$, $v = 0$, $a = -9.8$, $t = ?$.

$$0^2 = (5.0199...)^2 + 2(-9.8)(s) \Rightarrow s \approx 1.2857...$$

and so the maximum height P reaches above the ground is $3 + 3 + 1.285... = \boxed{7.3}$ m to 2 significant figures.

Question Sheet: **Sheet 10**

Model Solution No: 7

(a) $0.63 = 0.1 + 0.4 + x \Rightarrow x = 0.13$

$0.13 + 0.4 + 0.1 + y + 0.07 = 1 \Rightarrow y = 0.3$

Answer: e.g. $x = 0.13$, $y = 0.3$

(b) **Answer:** They are not independent.

Note that $P(M) = 0.63$ and $P(B) = 0.4$

Also note that probability of doing maths and biology is 0.1 from the Venn diagram.

But $0.4 \times 0.63 \neq 0.1$, and thus since $P(M) \times P(B) \neq P(M \text{ and } B)$, the events are not statistically independent.

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