CRASHMATHS
SOLUTIONS TO QUESTION COUNTDOWN

Question Sheet: **Sheet 1**

Model Solution No: 1

(a) **Answer:** $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 8x - \dfrac{1}{\sqrt{x}}$

Equivalent answers such as $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 8x - x^{-\frac{1}{2}}$ are also acceptable.

(b) **Answer:** $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8 + \dfrac{1}{2\sqrt{x^3}}$

Equivalent answers such as $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8 + \dfrac{1}{2}x^{-\frac{3}{2}}$ are also acceptable.

(c) **Answer:** $\displaystyle\int y \,\mathrm{d}x = \frac{4}{3}x^3 - \frac{4}{3}x^{\frac{3}{2}} + x + c$

Equivalent answers such as $\dfrac{4}{3}x^3 - \dfrac{4}{3}\sqrt{x^3} + x + c$ are also acceptable.

Question Sheet: **Sheet 1**

Model Solution No: 2

(a) Taking logs on both sides gives $\log(5^x) = \log(2)$.

Using the power rule, we then have $x \log(5) = \log(2)$ and so $x = \frac{\log(2)}{\log(5)}$.

Answer: $x = 0.431$ to three significant figures.

(b) We can combine the logs to get $\log_4 [2x(x-2)] = 2$.

Removing logs gives $2x(x-2) = 16$, which re-arranges to $x^2 - 2x - 8 = 0$.

Hence $(x-4)(x+2) = 0$. But we require $x > 2$ for both logs to be defined, so $x = 4$.

Answer: $x = 4$

Question Sheet: **Sheet 1**

Model Solution No: 3

(a) The gradient of the line is $m = \dfrac{4-2}{-1-4} = -\dfrac{2}{5}$.

Hence the equation of l is

$$y - 2 = -\frac{2}{5}(x-4)$$

using $y - y_1 = m(x - x_1)$. Re-arranging then gives the final result.

Answer: $y = -\dfrac{2}{5}x + \dfrac{18}{5}$

(b) Suppose they did intersect. Then the x coordinates of the intersection points satisfy

$$-\frac{2}{5}x + \frac{18}{5} = x^2 + 2x + p$$

Re-arranging gives

$$5x^2 + 12x + (5p - 18) = 0$$

Hence if they don't intersect, this equation can have no solutions.

This means the discriminant must be negative.

$$\begin{aligned}
&\Rightarrow \Delta = 12^2 - 4(5)(5p-18) < 0 \\
&\Rightarrow 144 - 100p + 360 < 0 \\
&\Rightarrow 100p > 504 \\
&\Rightarrow p > \frac{504}{100}
\end{aligned}$$

Answer: $p > \dfrac{126}{25}$

NB: 5.04 or other equivalent answers are acceptable.

Question Sheet: **Sheet 1**

Model Solution No: 4

With unstructured questions, it's good to plan the method. Here, we have two steps:

Step 1: Find the equation of the curve

Step 2: Integrate our equation of the curve between $x = 1$ and $x = 4$ to find the area.

Step 1.

The curve is quadratic and crosses the x axis at $x = 1$ and $x = 4$, so it must be of the form $y = k(x - 1)(x - 4)$.

We need the k there because we have to also ensure that the curve passes through the y axis at $y = 8$. For this to be achieved, we substitute $(0, 8)$ into the equation to find

$$8 = k(0 - 1)(0 - 4) \Rightarrow k = 2$$

Hence the equation of the curve C is $y = 2(x-1)(x-4)$, which expands to $y = 2x^2-10x+8$

Step 2.

Hence the signed area is

$$\begin{aligned}
\int_1^4 y \, \mathrm{d}x &= \int_1^4 (2x^2 - 10x + 8) \, \mathrm{d}x \\
&= \left[\frac{2}{3}x^3 - 5x^2 + 8x\right]_1^4 \\
&= \left(\frac{2}{3}(4)^3 - 5(4)^2 + 8(4)\right) - \left(\frac{2}{3}(1)^3 - 5(1)^2 + 8(1)\right) \\
&= -9
\end{aligned}$$

Hence the area of the shaded region is 9 units^2.

Answer: 9 units^2. [NB: answer of 9 alone is fine]

Question Sheet: **Sheet 1**

Model Solution No: 5

(a) Recall that $\cos(x)$ oscillates between 1 and -1. So the maximum value occurs when $\cos(15t) = -1$ to give $T_{\max} = 12 - 3(-1) = 15$. Similarly the minimum value is $T_{\min} = 12 - 3(1) = 9$.

Answer: Maximum = 15°C. Minimum = 9°C.

(b) We want to solve $12 - 3\cos(15t) = 10$ for $0 \leq t < 24$. Here is one way to solve:

Let $X = 15t$, then the equation becomes $12 - 3\cos X = 10$ for $0 \leq X < 360$.

This gives $\cos X = \frac{2}{3} \Rightarrow X_{\text{principal}} = 48.18....$

The other value of X in range is $360 - 48.18... = 311.81...$

Now we reverse our substitution to get: $15t = 48.18...$ or $311.81... \Rightarrow t = 3.21$ or $t = 20.8$ to 3 sf.

Answer: $t = 3.21$ or $t = 20.8$ to 3 sf.

(c) We want to add 273 to all of our values of T from the original model. So a refined model that does this is $T = 12 - 3\cos(15t) + 273 = 285 - 3\cos(15t)$.

Answer: $T = 285 - 3\cos(15t)$

Question Sheet: **Sheet 1**

Model Solution No: 6

Taking downwards as positive, we have

$$s = 3\text{ m}, \quad u = 0\text{ m s}^{-1}, \quad v = ?, \quad a = 9.8\text{ m s}^{-2}, \quad t = ?$$

(a) Using $v^2 = u^2 + 2as$, we have $v^2 = 2(9.8)(3) = 58.8$. Hence the speed when the ball hits the ground is $\sqrt{58.8} = 7.7\text{ m s}^{-1}$ to 2 sf.

Answer: 7.7 m s^{-1} to 2 sf. [Remember: we have used $g = 9.8$, so should give our answer to 2 or 3 significant figures.]

(b) Using $s = ut + \frac{1}{2}at^2$, we have

$$3 = \frac{1}{2}(9.8)t^2$$

which gives $t = 0.78$ s to 2 sf.

Answer: 0.78 s to 2 sf.

(c) The gradient of a velocity-time graph is acceleration. The problem involves a ball moving freely under the influence of gravity, so the magnitude of the gradient is the acceleration due to gravity $= 9.8\text{ m s}^{-2}$

Answer: 9.8 m s^{-2}.

(d) **Answer:** e.g. he assumed the ball bounces to the same height it was dropped / he assumed the ball does not lose energy when it hits the ground / the ground is smooth. (More advanced formulations, e.g. ‘elastic collisions with the ground’ are of course fine too.)

Question Sheet: **Sheet 1**

Model Solution No: 7

(a) **Answer:** It corresponds to a total daily sunshine reading that is greater than 24 hours, which is impossible.

(b) **Answer:** For every 1°C increase in temperature, the total daily sunshine increases by 0.297 hours.

(c) **Answer:** This is the regression line for s on t which should only be used to find values of s given t / should use the regression line for t on s instead.

(d) **Answer:** Pryia's claim must be wrong. It is not possible for the data to be associated with Perth, since there is no data on total daily sunshine for Perth (or any of the overseas locations).

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