Revision Session 5 – Solutions to Question Set 5i

Brief solutions to the Question Set. View the videos for the detailed worked solutions.

Question 1

(a) $x = \pm 2$
(b) $-5 < y < 5$
(c) We know that $-2 < z - 1 < 2$.
Adding one to both sides gives $-1 < z < 3$

Question 2

(a) https://www.desmos.com/calculator/jzy6hjke68

(b) Two approaches – you should understand both. Sometimes one is better than another:

Approach 1 (graphical):

Solutions given for when the line $y = 5$ intersects the line $y = |x - 1|$.
See https://www.desmos.com/calculator/qfbubbsx01

The positive solution is just given by $x - 1 = 5$ which gives $x = 6$
[Why: the straight line segment for positive x has the equation $y = x - 1$]

The negative solution is given by $-x + 1 = 5$, which gives $x = -4$
[Why: the straight line segment for negative x has the equation $y = -x + 1$]

Approach 2 (algebraic):

For the equation to be satisfied either $x - 1 = 5$ (because $|5| = 5$) or $x - 1 = -5$ (because $|-5| = 5$)

First eq gives $x = 6$. Second eq gives $x = -4$

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Question 3

(a) See https://www.desmos.com/calculator/unsacpuhxi. Note the x intersection is at $x = -a/3$ (because this when the modulus is 0 – it is the critical value if you like)

(b) https://www.desmos.com/calculator/ppvgrtxgxy

(c) The graphs only intersect once

(d) Always best to write in the form $|3x+2| = \frac{1}{x}$ (so modulus on one side)

Then either $3x+2 = \frac{1}{x}$ or $3x+2 = -\frac{1}{x}$.

It is only the first equation that is relevant, though (you can see this from the graph or find out by solving it and getting no solutions).

If you solve $3x+2 = \frac{1}{x}$ (multiply by x, factorise, etc) , you obtain

$x = 1/3$ or $x = -1$, but we discount the negative solution by looking at the graph.

So $x = 1/3$

{This question demonstrates the importance of being able to use graphical reasoning to complement your algebraic reasoning}

Question 4

$$|5^x - 1| = \frac{1}{3}(5^x)$$

So either $5^x - 1 = \frac{1}{3}(5^x)$ or $5^x - 1 = -\frac{1}{3}(5^x)$

$$5^x - 1 = \frac{1}{3}(5^x)$$
$$\Leftrightarrow \frac{2}{3}(5^x) = 1$$
$$\Leftrightarrow 5^x = \frac{3}{2}$$
$$\Leftrightarrow x\log 5 = \log\frac{3}{2}$$
$$\Leftrightarrow x = 0.252$$

You solve the other one in a similar way and find $x = -0.179$ (3sf)

Question 5

(a) (i) P is where $y = |2x + 7|$ intersects the x axis. This is at $2x + 7 = 0$, so $x = -7/2$

(ii) The curve $y = (x + k)^2$ meets the x axis also at $x = -7/2$ from the diagram. So we need $k = 7/2$.

[Can think about this in two ways.

Way 1: $y = (x + k)^2$ is a horizontal translation of $y = x^2$ by k units to the left. $y = x^2$ meets the x axis at 0, so we need to shift by $k = 7/2$ to get it to have its minimum point at $x = -7/2$.

Way 2: to meet x axis at $x = -7/2$, need $y = 0$ and $x = -7/2$.

So $0 = \left(-\frac{7}{2} + k\right)^2 \Rightarrow k = \frac{7}{2}$]

(b) Q and R are the intersection points of the parabola and the line.

Q is given by $2x + 7 = \left(x + \frac{7}{2}\right)^2$. This is a quadratic equation. If you expand out and factorise, you will find solutions are $x = -7/2$and $x = -1.5$. But for Q it is $x = -1.5$ (the other one corresponds to the point P).

Now for R either solve $2x + 7 = -\left(x + \frac{7}{2}\right)^2$ or note that Q and R are equally spaced either side of the turning point P of the parabola. So the x coordinate of R has to be $x = -5.5$

Substituting back in, you will find the y coordinate is 4 for both Q and R (which again is what you expect if you think about the symmetry of the problem)

So $Q\left(-\frac{3}{2}, 4\right)$, $R\left(-\frac{11}{2}, 4\right)$