Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				

Examiner Comments	Total Marks

MATHEMATICS

A LEVEL QUESTION COMPILATION

CM

Questions on: Sequences and Series

Instructions to candidates:

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer ALL of the questions.
- You must write your answer for each question in the spaces provided.
- You may use a calculator.

Information to candidates:

- Full marks may only be obtained for answers to ALL of the questions.
- The marks for individual questions and parts of the questions are shown in round brackets.
- There are 14 questions in this question paper. The total mark for this paper is 106.

Advice to candidates:

- You should ensure your answers to parts of the question are clearly labelled.
- You should show sufficient working to make your workings clear to the Examiner.
- Answers without working may not gain full credit.

A2/QS/6

1 5 3 3 2 2 2 1 8 0 0 0 4

1 A sequence of numbers $x_1, x_2, x_3, \ldots$ is defined such that

$$x_{n+1} = 4x_n - 1, \quad x_1 = 4$$

(a) Find x_2 and x_3. **(2)**

(b) Evaluate $\sum_{n=1}^{4} x_n$. **(2)**

Do not write outside the box

Question 1 continued

TOTAL 4 MARKS

2 A sequence of numbers $x_1, x_2, x_3, \ldots$ is defined such that

$$x_{n+1} = 4x_n + k, \quad \text{where } k \text{ is a constant}$$

Given that $x_1 = 6$,

(a) find, in terms of k, x_2 and x_3. **(2)**

(b) Determine the value of k in the case where $\sum_{n=1}^{4} x_n = 429$. **(3)**

Do not write outside the box

Question 2 continued

TOTAL 5 MARKS

3 A sequence of numbers $x_1, x_2, x_3, \ldots$ is defined such that

$$x_{n+1} = 2x_n - k, \quad x_1 = 20$$

Given that $x_3 = 50$,

(a) find the value of the constant k. **(2)**

(b) Hence evaluate

(i) $\sum_{n=1}^{4} x_n$

(ii) $\sum_{n=1}^{4} (2x_n - 1)$

(3)

Do not write outside the box

Question 3 continued

TOTAL 5 MARKS

4 A sequence of numbers $x_1, x_2, x_3, \ldots$ is defined such that

$$x_{n+1} = 2x_n - k, \quad x_1 = 20$$

Given that $x_3 = 50$,

(a) find the value of the constant k. **(2)**

(b) Hence evaluate

(i) $\sum_{n=1}^{4} x_n$

(ii) $\sum_{n=1}^{4} (2x_n - 1)$

(3)

Question 4 continued

TOTAL 5 MARKS

5 A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined such that

$$u_{n+1} = ku_n + 5, \quad u_1 = 1$$

Given that the sequence is a periodic sequence with order 2,

(a) find the possible values of k. **(3)**

(b) Hence, find $\sum_{n=1}^{100} u_n$ for each of the values of k found in (a). **(3)**

Question 5 continued

TOTAL 6 MARKS

6 Mark begins to save money each week. In week 1, he saves £30, in week 2, he saves £35, in week 3, he saves £40, and so on, such that the amount of money he saves each week forms an arithmetic sequence.

(a) Find the amount of money Mark saves in Week 10. **(2)**

(b) Find the total amount of money Mark has saved by Week 12. **(2)**

Mark is saving for a computer which costs £1200.

(c) Determine the minimum number of weeks Mark must save for so that he has enough money to purchase his computer. **(3)**

Do not write outside the box

Question 6 continued

TOTAL 7 MARKS

7 An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of this series is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(4)

Linda repays a loan over a period of n months. Her monthly repayments form an arithmetic sequence.

She repays £200 in the first month, £195 in the second month, £190 in the third month, and so on. She makes her final repayment in the n th month, where $n > 18$.

(b) Find the amount Linda repays in the 18th month. **(2)**

Over the n months, Linda repays a total of £3500.

(c) By forming and solving an equation in n, find the value of n.
Show your working clearly. **(4)**

Do not write outside the box

Question 7 continued

TOTAL 10 MARKS

8 An arithmetic sequence has first term a and common difference d.

The 3rd term in this sequence is 2.

The sum of the first 10 terms of the sequence is 95.

Showing your method clearly, find the value of a and the value of d. **(5)**

Question 8 continued

TOTAL 8 MARKS

9 (a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2+4+6+\ldots+100$$

(3)

(b) Consider the following arithmetic sequence

$$(2k+3),\ (5k+5),\ (8k+7),\ \ldots,$$

The 50th term of this sequence is 250.

Find the sum of the first 50 terms in this sequence. **(5)**

Do not write outside the box

Question 9 continued

TOTAL 8 MARKS

10 The first term of a geometric series is 150. The sum to infinity of the series is 225.

(a) Find the common ratio of this series. **(3)**

(b) Calculate

(i) the 6th term in the series, **(2)**

(ii) the sum of the first 8 terms of the series. **(2)**

The sum of the first *n* terms of the series is greater than 200.

(c) Showing your method clearly, find the smallest value of *n*. **(4)**

Do not write outside the box

Question 10 continued

TOTAL 11 MARKS

11 A geometric series has first term a and common ratio r.

(i) Prove that the sum of the first n terms of the series is

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

(ii) The sum of the first 8 terms of another geometric series is 12.

The sum to infinity is 16.

Find the the first term and common ratio of this series. **(4)**

Do not write outside the box

Question 11 continued

TOTAL 8 MARKS

12 Erwin is completing a number of experiments using a chemical. In each experiment, he must increase the amount of chemical he uses. In the first experiment, he uses 7 g of the chemical and in the second experiment, he uses 8.2 g of the chemical.

Two models, A and B, are created for the amount of chemical used.

In model A, the amount of chemical used is assumed to form an arithmetic progression.

(a) Use model A to find the total amount of chemical Erwin uses in the first 25 experiments. **(3)**

In model B, the amount of chemical used in assumed to form a geometric progression.

(b) Use model B to find the total amount of chemical Erwin uses in the first 25 experiments. **(3)**

There is a total of 1800 g of the chemical available to Erwin.

(c) Showing your working clearly, determine the greatest number of experiments possible in each of the two models. **(6)**

Question 12 continued

TOTAL 12 MARKS

13 The adult population of a town is 30 000 at the end of Year 1.

A model predicts that the population of the town will increase by 3% each year, forming a geometric sequence.

(a) Find the size of the adult population in the town at the end of Year 4. **(3)**

The model predicts that Year N will be the first year in which the adult population of the town exceeds 42 000.

(b) Show clearly that $(N-1)\log 1.03 > \log 30$. **(3)**

(c) Hence find the value of N. **(2)**

(d) State **one** limitation of the model. **(1)**

Question 13 continued

TOTAL 9 MARKS

14 (a) State the condition for an infinite geometric series with first term a and common ratio r to be convergent. **(1)**

(b) (i) Find $\sum_{r=1}^{10} 2(3^r)$. **(2)**

(ii) Does $\sum_{r=1}^{\infty} 2(3^r)$ converge? Explain your reasoning. **(2)**

(c) Show that the series $\sum_{n=1}^{\infty} e^{-nx}$ converges **and** find its value. **(3)**

Question 14 continued

TOTAL 8 MARKS