Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				

Examiner Comments	

Total Marks

MATHEMATICS

A LEVEL PAPER 1

CM

Bronze Set C (Edexcel Version)

Time allowed: 2 hours

Instructions to candidates:

• In the boxes above, write your centre number, candidate number, your surname, other names and signature.

• Answer ALL of the questions.

• You must write your answer for each question in the spaces provided.

• You may use a calculator.

Information to candidates:

• Full marks may only be obtained for answers to ALL of the questions.

• The marks for individual questions and parts of the questions are shown in round brackets.

• There are 13 questions in this question paper. The total mark for this paper is 100.

Advice to candidates:

• You should ensure your answers to parts of the question are clearly labelled.

• You should show sufficient working to make your workings clear to the Examiner.

• Answers without working may not gain full credit.

1 0 3 3 3 1 3 2 8 0 0 0 4

1 Find an equation of the tangent to the curve

$$x^4 - 4x^2y + y^4 = 49$$

at the point (2, 3). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found. **(6)**

Question 1 continued

TOTAL 6 MARKS

2 Prove by exhaustion that

$$1^3 + 2^3 + \ldots + n^3 \equiv (1 + 2 + \ldots + n)^2$$

for positive integers 1 to 6 inclusive. **(3)**

Question 2 continued

TOTAL 3 MARKS

3 The equation of the curve C satisfies

$$\frac{dy}{dx} = \frac{2 - x^2}{x}, \quad x > 0$$

Given that the point $(1, 4)$ lies on C, express y in terms of x. **(5)**

Question 3 continued

TOTAL 5 MARKS

4

$$g(x) = \frac{1}{\sqrt{(4+3x)}}, \quad |x| < \frac{3}{4}$$

(a) Find, in ascending powers of x, the first 3 terms in the binomial expansion of $g(x)$, up to and including the term in x^2. Give each term in its simplest form. **(5)**

(b) Find the value of $\frac{1}{\sqrt{(4+3x)}}$ when $x = \frac{1}{3}$.

Give your answer in the form $k\sqrt{5}$. **(1)**

(c) Substitute $x = \frac{1}{3}$ into your binomial expansion in part (a) and hence find an approximation for the value of $\sqrt{5}$. Give your approximation as a simplified fraction. **(2)**

Question 4 continued

TOTAL 8 MARKS

5

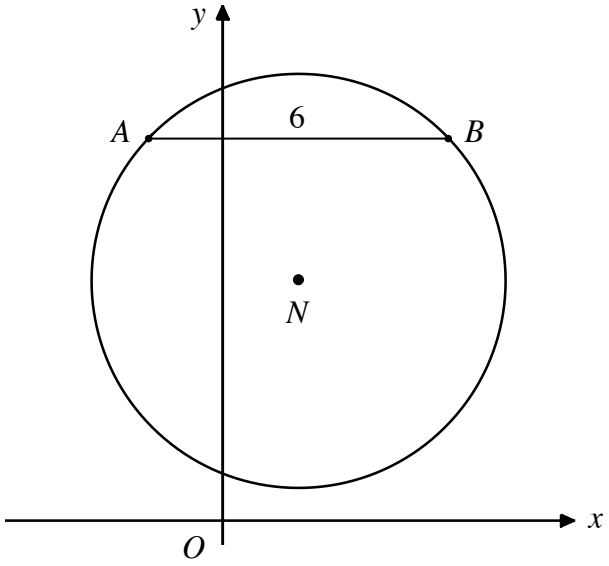

Figure 1

Figure 1 above shows a circle with equation

$$x^2 + y^2 - 2x - 10y - 8 = 0$$

The circle has centre N and radius r.

(a) Find the coordinates of N and the value of r. **(4)**

The points A and B lie on the circle such that the chord AB is parallel to the x axis and has length 6 units, as shown in Figure 1 above.

(b) Find the coordinates of A and the coordinates of B. **(3)**

Question 5 continued

TOTAL7 MARKS

6 Given that

$$f(x) = 5e^{x} - 2, \quad x \in \mathbb{R}$$

(a) sketch, on separate axes, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each sketch, show clearly the coordinates of any points where the curve crosses or meets the coordinate axes **and** state the equations of any asymptotes. **(6)**

(b) Find the exact solutions of the equation $|f(x)| = 1$. **(3)**

(c) Write down the **set** of values of k for which the equation

$$|f(x)| = k$$

has one real solution for x. **(1)**

Question 6 continued

TOTAL 10 MARKS

7 A sequence is defined such that

$$x_n = (-1)^n, \quad n \geqslant 1$$

(a) Show that the sequence is periodic **and** state its order. **(2)**

(b) Given an integer $k > 1$, state the value of $\sum_{n=1}^{k} x_n$ in the case where

(i) k is odd,

(ii) k is even. **(2)**

Question 7 continued

TOTAL 4 MARKS

8 (a) Prove that

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}\equiv 2\operatorname{cosec}\theta,\quad \theta\neq n\pi,\ n\in\mathbb{Z}$$

(4)

(b) Hence solve, for $0 < x \leqslant \pi$,

$$\frac{\sin 3x}{1+\cos 3x}+\frac{1+\cos 3x}{\sin 3x}=\sqrt{5}$$

Give your answers to 2 decimal places. **(5)**

Question 8 continued

TOTAL 9 MARKS

9 Theo is studying a population of rabbits. The population p at the time t weeks after the study started is modelled to be

$$p = 0.07t^2 + 4t + 110, \quad t \geqslant 0$$

(a) Write down the size of the population at the start of the study. **(1)**

(b) Show that, according to this model, the size of the population will increase over time. **(2)**

Theo creates a second model for how p varies with t, which is given by

$$p = \frac{440a\,e^{0.1t}}{1 + a\,e^{0.1t}}, \quad t \geqslant 0$$

(c) Using your answer to part (a), show that $a = \frac{1}{3}$. **(3)**

(d) Hence use the second model to determine the time taken for the population to reach a size of 300 rabbits. **(3)**

(e) Determine the long-term behaviour of Theo's two models **and** hence suggest which model is best for Theo's study. **(3)**

Question 9 continued

Question 9 continued

Question 9 continued

TOTAL 12 MARKS

10 The curve C has the equation

$$x = 4\tan(3y)$$

(a) Show that the point $P\left(4\sqrt{3},\ \frac{\pi}{9}\right)$ lies on C. **(1)**

(b) Find $\frac{dx}{dy}$ and hence show that $\frac{dy}{dx} = \frac{4}{48 + 3x^2}$. **(3)**

(c) Find the equation of the normal to C at the point P, giving your answer in the form $y = mx + c$, where m and c are exact constants to be found. **(4)**

Question 10 continued

TOTAL 8 MARKS

11

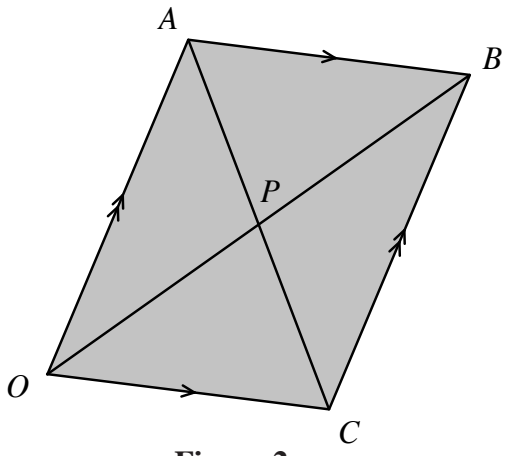

Figure 2

Figure 2 above shows a parallelogram $OABC$.

Relative to the fixed origin O, the position vector of A is $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and the position vector of C is $2\mathbf{i} - \mathbf{j} + \mathbf{k}$.

(a) Find a unit vector in the direction $\overrightarrow{OB}$. **(3)**

The lines OB and AC are the diagonals of the parallelogram.
The lines intersect at the point P.

(b) Use vectors to prove that the diagonals of the parallelogram bisect each other. **(4)**

(c) By using the cosine rule on triangle OPC, or otherwise, find the angle OPC. **(4)**

Question 11 continued

Question 11 continued

Question 11 continued

TOTAL 11 MARKS

12 (a) Prove that the square root of 6 is irrational. **(4)**

Irini wants to prove that $\sqrt{2}+\sqrt{3}$ is irrational.

She claims that: "Since $\sqrt{2}$ is irrational and $\sqrt{3}$ is irrational, $\sqrt{2}+\sqrt{3}$ is also irrational."

(b) Explain why Irini's reasoning is **incorrect**.

In your answer, you should state the assumption she has made and use a suitable counterexample to disprove her assumption. **(2)**

Jack provides a correct proof of the fact that $\sqrt{2}+\sqrt{3}$ is irrational.

Part of his proof is shown below.

Assume $\sqrt{2}+\sqrt{3}=r$, where r is rational

$\Rightarrow r^2=(\sqrt{2}+\sqrt{3})^2$

$\Rightarrow r^2=5+2\sqrt{6}$

$\Rightarrow \sqrt{6}=\dfrac{r^2-5}{2}$

which is a contradiction because ...

(c) Complete the final line of Jack's proof to explain why he has obtained a contradiction. **(1)**

Question 12 continued

Question 12 continued

Question 12 continued

TOTAL 7 MARKS

13

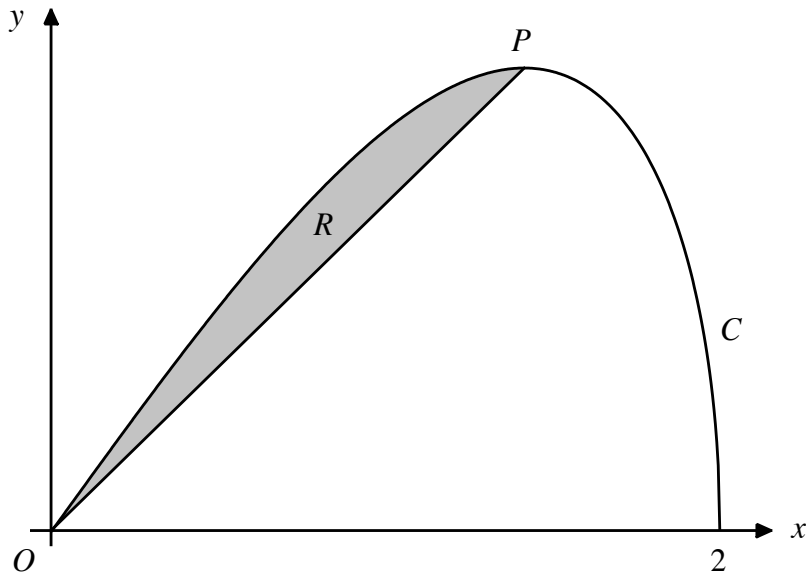

Figure 3

Figure 3 above shows a sketch of the curve C with equation $y = x\sqrt{4 - x^2}$ for $0 \leqslant x \leqslant 2$. The point P is a maximum point on C. The region R, shown shaded in the figure, is bounded by the curve C and the line segment OP, where O is the origin.

Showing your method clearly, find the exact area of R.
Give your answer in the form $a + b\sqrt{2}$, where a and b are rational numbers to be found. **(10)**

Question 13 continued

Question 13 continued

Question 13 continued

END OF PAPER **TOTAL 10 MARKS**

TOTAL FOR PAPER IS 100 MARKS