1. $$\frac{x^2 + 4\sqrt{x}}{x} = \frac{x^2}{x} + \frac{4x^{1/2}}{x} = x + 4x^{-1/2}$$

$$\boxed{p = 1,\ q = -1/2.}$$

2. (a) $$8^{2x+1} = (2^3)^{2x+1}$$
$$= 2^{3(2x+1)}$$
$$= 2^{6x+3}$$, so $\underline{y = 6x+3}$

(b) want to solve

$$2^{6x+3} = 4^x$$
$$2^{6x+3} \overset{!}{=} 2^{2x}$$
$$\Rightarrow 6x+3 = 2x$$
$$\Rightarrow -4x = 3$$
$$\Rightarrow \boxed{x = -3/4}$$

3. (a) $$\frac{(2x^{1/2})^3}{4x^2} = \frac{8x^{3/2}}{4x^2} = \underline{2x^{-1/2}}.$$; $a = 2$, $p = -1/2$

(b) $$2x^{-1/2} = 8 \Rightarrow x^{-1/2} = 4$$
$$\Rightarrow \frac{1}{x^{1/2}} = 4$$
$$\Rightarrow x^{1/2} = 1/4$$
$$\Rightarrow x = 1/16.$$

4. Eq. is

$$\cancel{2} \; 3^{2x+2} - 244(3^x) + 27 = 0$$

$$\Rightarrow 3^2 \cdot 3^{2x} - 244(3^x) + 27 = 0$$

$$\Rightarrow 9 \cdot (3^{2x}) - 244(3^x) + 27 = 0$$

$$\Rightarrow 9y^2 - 244y + 27 = 0$$

$$\Rightarrow (9y - 1)(y - 27) = 0$$

$$\Rightarrow y = 1/9 \quad \text{or} \quad y = 27$$

So $3^x = 1/9$ or $3^x = 27$

By inspechon, $\underline{x = -2 \quad \text{or} \quad 3.}$

5. (a) $(1-\sqrt{8})^2 = (1-\sqrt{8})(1-\sqrt{8})$

$$= 1 - \sqrt{8} - \sqrt{8} + 8$$

$$= 9 - 2\sqrt{8}.$$

$$= 9 - 2(2\sqrt{2})$$

$$= \underline{9 - 4\sqrt{2}}, \quad a = 9, \; b = -4.$$

(b) $$\frac{2-\sqrt{2}}{1+\sqrt{2}} = \frac{(2-\sqrt{2})}{(1+\sqrt{2})} \times \frac{(1-\sqrt{2})}{(1-\sqrt{2})}$$

$$= \frac{(2-\sqrt{2})(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})}$$

$$= \frac{2 - 2\sqrt{2} - \sqrt{2} + 2}{1 - 2}$$

$$= \frac{4 - 3\sqrt{2}}{-1}$$

$$= \underline{3\sqrt{2} - 4.} \quad , \quad a = -4, \; b = 3.$$

6. $$\text{Area} = \frac{1}{2} \times |AB| \times |BC|$$

$$(8 + 5\sqrt{3}) = \frac{1}{2} \times (4 + 2\sqrt{3}) \times |BC|$$

$$\Rightarrow |BC| = \frac{2(8 + 5\sqrt{3})}{4 + 2\sqrt{3}}$$

$$= \frac{8 + 5\sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{(8 + 5\sqrt{3})(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= \frac{16 - 8\sqrt{3} + 10\sqrt{3} - 5(3)}{4 - 3}$$

$$= 1 + 2\sqrt{3}.$$

so $$|AC|^2 = |AB|^2 + |BC|^2$$

$$= (4 + 2\sqrt{3})^2 + (1 + 2\sqrt{3})^2$$

$$= 16 + 16\sqrt{3} + 12 + 1 + 4\sqrt{3} + 12$$

$$= 41 + 20\sqrt{3}$$

so $$AC = \sqrt{\underset{41}{\cancel{31}} + 20\sqrt{3}} = (\overset{41}{\cancel{31}} + 20\sqrt{3})^{1/2}$$

7. $a^{1/2} + \sqrt{4}\sqrt{a} = 3$

$\Rightarrow a^{1/2} + 2a^{1/2} = 3$

$\Rightarrow 3a^{1/2} = 3$

$\Rightarrow a^{1/2} = 1$

$\Rightarrow \underline{a = 1.}$

8. $x - 2y = 1 \qquad (1)$

$x^2 + y^2 = 29 \qquad (2)$

$(1) \Rightarrow x = 2y + 1$

So $(2y+1)^2 + y^2 = 29$

$\Rightarrow 4y^2 + 4y + 1 + y^2 = 29$

$\Rightarrow 5y^2 + 4y - 28 = 0$

$\Rightarrow (5y + 14)(y - 2) = 0$

$\Rightarrow y = -14/5$ or $y = 2$

when $y = 2$, $x = 2(2) + 1 = 5.$

when $y = -14/5$, $x = 2\left(-\frac{14}{5}\right) + 1$

$= -\frac{28}{5} + 1$

$= -\frac{23}{5}.$

so solutions are

$$\boxed{\begin{array}{c} x = 5,\quad y = 2 \\ \text{or} \\ x = -\frac{23}{5},\quad y = -14/5 \end{array}}$$

9. $x^2 + 4y^2 = 1 \quad (1)$

$2y = x + 1 \quad (2)$

Put (2) into (1): $x^2 + (x+1)^2 = 1$

$\Rightarrow x^2 + x^2 + 2x = 0$

$\Rightarrow 2x^2 + 2x = 0$

$\Rightarrow 2x(x+1) = 0$

$\Rightarrow x = 0$ or $x = -1$.

when $x = 0$, $2y = 1 \Rightarrow y = 1/2$

when $x = -1$, $2y = 0 \Rightarrow y = 0$

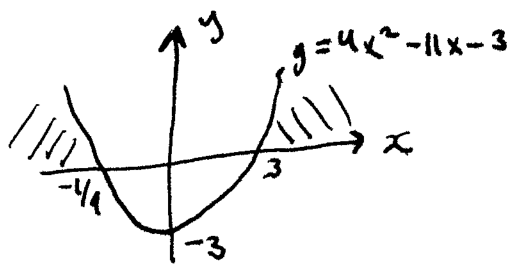
solutions are $\boxed{x = 0,\ y = 1/2 \text{ or } x = -1,\ y = 0}$

10. $4x^2 - 11x - 3 > 0$

Critical values given by $4x^2 - 11x - 3 = 0$

$\Rightarrow (\cancel{4}x - 3)(4x + 1) = 0$

$\Rightarrow x = 3$ or $x = -1/4$.



so $4x^2 - 11x - 3 > 0$ when

$x < -1/4$ or $x > 3$

$\therefore$ in set notation, solutions are $\{x \in \mathbb{R} : x < -1/4 \text{ or } x > 3\}$.

11. (a) $5 - 4x > 1 \iff -4x > -4$

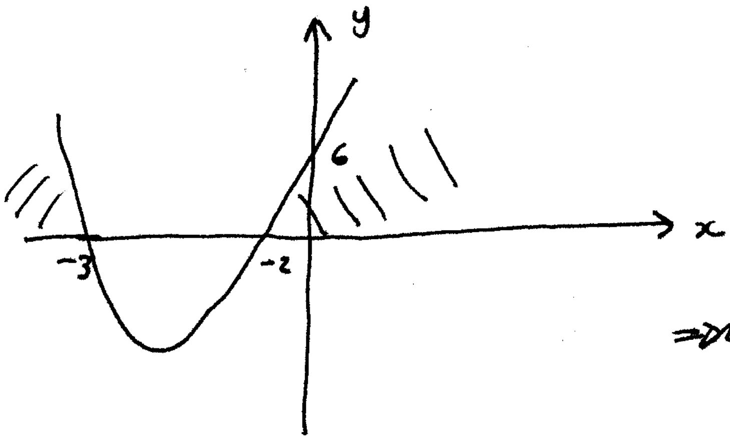
$\iff \underline{x < 1}.$

(b) $x^2 + 5x + 6 > 0$

Critical values given by $x^2 + 5x + 6 = 0$

$\Rightarrow (x+3)(x+2) = 0$

$\Rightarrow x = -3$ or $x = -2$.



So solutions are

$\underline{x > -2 \text{ or } x < -3}$

~~$\Rightarrow$ in set notation~~

(c) Need to satisfy $\underline{\text{both}}$ $x < 1$

$\underline{\text{and}}$ $x > -2$ or $x < -3$

So either $x < -3$ or $-2 < x < 1$

in set notation, this is

$\underline{\{x \in \mathbb{R} : x < -3\} \cup \{x \in \mathbb{R} : -2 < x < 1\}}.$

<u>Two methods:</u>

12) $\frac{3+x}{x} > 3$ (multiply by x^2).

$(3+x)x > 3x^2$

$\Rightarrow 3x + x^2 > 3x^2.$

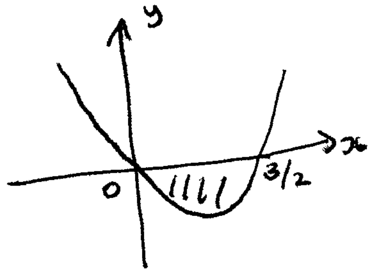
$\Rightarrow \underline{2x^2 - 3x < 0}$

$\cancel{\Rightarrow}$

Critical values $2x^2 - 3x = 0$

$x(2x-3) = 0$

so $x = 0$ or $x = 3/2$.



$0 < x < 3/2.$

As a set, solutions are

$\{x \in \mathbb{R} : 0 < x < 3/2\}.$

<u>Method 2:</u> <u>consider cases:</u>

If $x > 0$, then $\frac{3+x}{x} > 3 \Rightarrow 3 + x > 3x$

$\Rightarrow 2x < 3$

$\Rightarrow x < 3/2.$

If $x < 0$, then $\Rightarrow 3 + x < 3x \Rightarrow \cancel{x}\ 2x > 3$ (# impossible since $x < 0$).

so $\underline{0 < x < 3/2}$ (or in set notation $\{x \in \mathbb{R} : 0 < x < 3/2\}$.

13. (a) $f(x) = (x+1)^2 - 1 + 5$

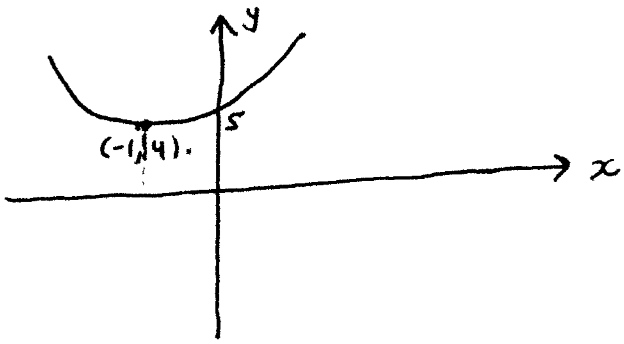
$= (x+1)^2 + 4$, $a = 1$, $b = 4$.

(b) $\Delta = 2^2 - 4(1)(5)$

$= 4 - 20$

$= -16 \ (< 0)$.

(c).



14. (a) $5x^2 - 15x + 6 = 5(x^2 - 3x) + 6$

$= 5\left[(x - 3/2)^2 - 9/4\right] + 6$

$= 5(x - 3/2)^2 - 45/4 + 6$

$= 5(x - 3/2)^2 - 21/4$.

(b) (i) Turning point $= (3/2, -21/4)$.

(ii) $5x^2 - 15x + 6 = 0$

$\Rightarrow 5(x - 3/2)^2 - 21/4 = 0$

$\Rightarrow 5(x - 3/2)^2 = 21/4$

$\Rightarrow (x - 3/2)^2 = 21/20$

$\Rightarrow x = 3/2 \pm \sqrt{21/20}$

15. (a) Equal roots $\Rightarrow \quad \Delta = 0$

$$\Rightarrow (3-k)^2 - 4(k)(-4) = 0$$

$$\Rightarrow 9 - 6k + k^2 + 16k = 0$$

$$\Rightarrow k^2 + 10k + 9 = 0$$

$$\Rightarrow (k+9)(k+1) = 0$$

$$\Rightarrow k = -1 \text{ or } k = -9.$$

(b) Two distinct roots $\Rightarrow \Delta > 0$.

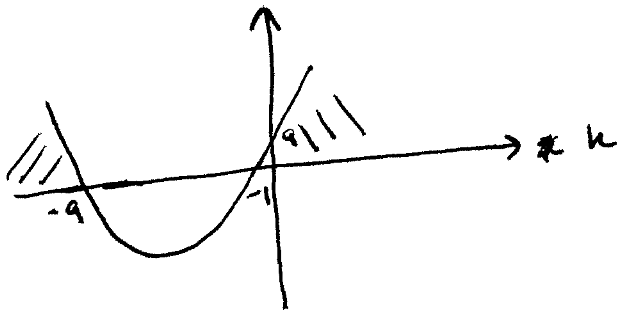
$$\Rightarrow (3-k)^2 - 4k(-4) > 0$$

$$\Rightarrow k^2 + 10k + 9 > 0.$$

Critical values are $k = -1$, $k = -9$.

So

$k < -9$ or $k > -1$.



As a set $\{k \in \mathbb{R} : k < -9 \text{ or } k > -1\}$.

16. (a) $x^2 + 2x + 1 = k(x^2 + 4)$

$\Rightarrow x^2 + 2x + 1 = kx^2 + 4k$

$\Rightarrow x^2 - kx^2 + 2x + 1 - 4k = 0$

$\Rightarrow x^2(1-k) + 2x + (1-4k) = 0$

No real roots $\Rightarrow \Delta < 0$

$\Rightarrow 2^2 - 4(1-k)(1-4k) < 0$

$\Rightarrow 4 - 4(1 - 5k + 4k^2) < 0$

$\Rightarrow 4 - 4 + 20k - 16k^2 < 0$

$\Rightarrow 20k - 16k^2 < 0$

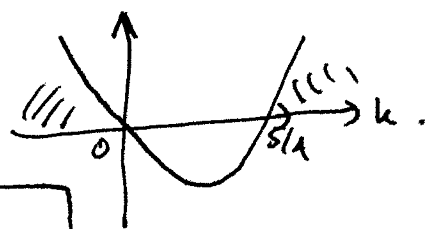
$\Rightarrow 16k^2 - 20k > 0$

$\Rightarrow \underline{4k^2 - 5k > 0}$, as required.

(b). Critical values are given by $4k^2 - 5k = 0$

$k(4k - 5) = 0$

so critical values are $k = 0$ or $k = 5/4$



$k < 0$, $\underline{k > 5/4}$.

$\boxed{\{k \in \mathbb{R} : k < 0\} \cup \{k \in \mathbb{R} : k > 5/4\}}$

(or $\{k \in \mathbb{R} : k < 0$ or $k > 5/4\}$).

(c) $k = 2$, so $\cancel{\#}$ 0 real roots.

$\Rightarrow$ <u>no intersections</u> with the x-axis.

17. (a) Total amount of fence $= 270$

$\Rightarrow \quad 3w + 2l = 270$

$\Rightarrow 3w = 270 - \cancel{3l}\ 2l$

$\Rightarrow \quad w = \dfrac{270-2l}{3} = \dfrac{270}{3} - \dfrac{2l}{3} = 90 - \dfrac{2l}{3}$.

as required.

(b). Area = length $\times$ width

$\Rightarrow \quad A = lw$

$= l(90 - 2l/3)$

$= 90l - \dfrac{2l^2}{3}$.

(c) First complete the square on A:

$A = -\dfrac{2l^2}{3} + 90l$

$= -\dfrac{2}{3}(l^2 - 135l)$

$= -\dfrac{2}{3}\left[(l - \frac{135}{2})^2 - (\frac{135}{2})^2\right]$ (by completing the square)

$= -\dfrac{2}{3}\left[(l - \frac{135}{2})^2 - \frac{18225}{4}\right]$

$= -\dfrac{2}{3}(l - \frac{135}{2})^2 + \dfrac{6075}{2}$

(i) so maximum area is $\dfrac{6075}{2}$ m²

(ii) max. occurs when $\boxed{l = \frac{135}{2}}$ and so when $\boxed{w = 90 - \frac{2}{3}\left(\frac{135}{2}\right) = 45}$