AS Level
Maths

Gold Set A, Paper 1 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1 (a)	$\overrightarrow{AC} = (\mathbf{i}+8\mathbf{j})-(-4\mathbf{i}-2\mathbf{j}) = 5\mathbf{i}+10\mathbf{j}$ $\Rightarrow \overrightarrow{AB} = \frac{2}{5}\overrightarrow{AC} = 2\mathbf{i}+4\mathbf{j}$ So $\overrightarrow{OB} = \overrightarrow{OA}+\overrightarrow{AC}$ $= (-4\mathbf{i}-2\mathbf{j})+(2\mathbf{i}+4\mathbf{j})$ $= -2\mathbf{i}+2\mathbf{j}$	M1 M1 A1 **[3]**	Complete method to find $\overrightarrow{AB}$ or $\overrightarrow{CB}$ (oe) Complete method to use their $\overrightarrow{AB}$ to find the position vector of B (or equivalent method using their vector) Correct position vector of B
1 (b)	Distance between O and D equals distance between B and C $\Rightarrow \lvert OD\rvert = \sqrt{(-2-1)^2+(2-8)^2} = 3\sqrt{5}$	M1 A1 oe **[2]**	Complete method to find the distance from O to D: so needs to find BC and then employ correct method to find the distance Correct distance oe, e.g. $\sqrt{45}$ or awrt 6.71 **ALT:** if they attempt to find D, M1 is for a correct method employed to find D followed by correct method used to find the distance
2 (a)	$y = x^{\frac{3}{2}} + kx^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - \frac{k}{2}x^{-\frac{3}{2}}$ $\left.\frac{dy}{dx}\right\rvert_{x=4} = 0 \Rightarrow \frac{3}{2}(4)^{\frac{1}{2}} - \frac{k}{2}(4)^{-\frac{3}{2}} = 0$ $\Rightarrow \frac{3}{2}(2) - \frac{k}{2}\left(\frac{1}{8}\right) = 0$ $\Rightarrow k = 48$ Hence at P, $y = \sqrt{4^3} + \frac{48}{\sqrt{4}} = 32$	M1* A1 M1(dep*) A1 A1ft **[5]**	Writes y as powers of x and attempts to find dy/dx ($x^n \to nx^{n-1}$) Correct derivative Uses that $x = 4$ is a stationary point and attempts to solve for k Correct value of k Correct y coordinate of P ft their k

2 (b)	$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}-24x^{-\frac{3}{2}}$ Stationary points occur when $\frac{3}{2}x^{\frac{1}{2}}-24x^{-\frac{3}{2}}=0 \quad \Rightarrow x^2=16$ which only has one solution for x since $x>0$. Hence P is the only stationary point on C $\frac{d^2y}{dx^2}=\frac{3}{4}x^{-\frac{1}{2}}+36x^{-\frac{5}{2}}$ so at $x=4$, $\frac{d^2y}{dx^2}=\frac{3}{4}(4)^{-\frac{1}{2}}+36(4)^{-\frac{5}{2}}=\frac{3}{2}>0$ Hence since $\frac{d^2y}{dx^2}>0$ at $x=4$, P is a minimum	M1 A1 M1 A1 **[4]**	Complete method to show that the curve has only one stationary point Convincingly shows that the curve only has one stationary point + conclusion Complete method to determine the nature of the stationary point Way 1 (scheme): computes second derivative and ***EITHER*** calculates its value at $x=4$ ***OR*** implies its sign Way 2: calculates value of y **or** derivative on either side of 4 States 'minimum' following a calculation, valid reason and correct conclusion
3 (a)	$c=2^m \;,\; d=16^n$ $\Rightarrow \frac{c^3}{\sqrt{d}}=\frac{2^{3m}}{16^{\frac{n}{2}}}$ $=\frac{2^{3m}}{2^{2n}}$ $=2^{3m-2n}$ (so $y=3m-2n$)	B1 B1 M1 A1 **[4]**	Correct expression for c in terms of m (can be implied) Correct expression for d in terms of n (can be implied) Complete method to simplify the quantity into a single term of the form 2^y with y in terms of m and n only using their c and d Obtains correct result
3 (b)	$\log(4^x\times5^{2-x})=\log7^{3-2x}$ $\Rightarrow \log4^x+\log5^{2-x}=\log7^{3-2x}$ $\Rightarrow x\log4+(2-x)\log5=(3-2x)\log7$ $\Rightarrow x\log4+2\log5-x\log5=3\log7-2x\log7$ $\Rightarrow x(\log4-\log5+2\log7)=3\log7-2\log5$ $\Rightarrow x=\frac{3\log7-2\log5}{\log4-\log5+2\log7}=\underline{\mathbf{0.714}}$ (3 sf)	M1[1] M1[2](dep1) M1[3](dep2) A1 cao **[4]**	Takes logs to both sides Uses product rule to expand out term on LHS Uses power rule to obtain linear equation in x Obtains correct value of x to 3 sf

<table>
<tr><td>4 (a)</td><td>Discriminant = $(\sqrt{8})^2 - 4(2\sqrt{2}-2)(1+\sqrt{2})$

$= 8 + 8(1-\sqrt{2})(1+\sqrt{2})$
$= 8 + 8(1 - 2)$
$= 8 - 8$
$= 0$
Hence the equation has two equal roots / a repeated root</td><td>M1*

M1(dep*)

A1
[3]</td><td>Writes down the correct discriminant in any form

Attempts to expand the brackets with correct surd manipulation seen

Obtains discriminant = 0 + conclusion</td></tr>
<tr><td>4 (b)</td><td>$x = \dfrac{-\sqrt{8} \pm 0}{2(2\sqrt{2}-2)}$
$= \dfrac{\sqrt{8}}{4(1-\sqrt{2})} \times \dfrac{1+\sqrt{2}}{1+\sqrt{2}}$
$= \dfrac{\sqrt{8}}{4(1-\sqrt{2})} \times \dfrac{1+\sqrt{2}}{1+\sqrt{2}}$
$= \dfrac{\sqrt{8}+4}{-4}$
=
$= -1 + \dfrac{1}{4}(2\sqrt{2})$
$= -1 - \dfrac{1}{2}\sqrt{2}$</td><td>M1*

M1(dep*)

A1
[3]</td><td>Writes down <u>correct</u> value of x in unsimplified form
May use completing the square in which case we still need them to reach $x = \ldots$ correctly
Attempts to simplify their expression by rationalising the denominator and using $\sqrt{8} = 2\sqrt{2}$

Correct value of x</td></tr>
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5 (a)	$0 \le \sin^2 x \le 1$ $\Rightarrow -3 \le -3\sin^2 x \le 0$ $\Rightarrow 2 \le 5 - 3\sin^2 x \le 5$ Hence $5 - 3\sin^2 x > 0$	M1 A1 **[2]**	Attempts to find a bound on $5 - 3\sin^2 x$ Alt: may use calculus Shows the result convincingly
5 (b)	$5 - 3\sin^2\theta = 4$ $\Rightarrow \sin^2\theta = \frac{1}{3}$ $\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}}$ $\Rightarrow \theta = \pm 35.264...$ Other values of θ given by 180 – 35.26 = 144.73… 180 + 35.26 = 215.26… 360 – 35.26 = 324.74… So θ = 35.3, 144.7, 215.3 or 324.7	M1* A1 M1 A1 A1 **[5]**	Removes logs correctly from both sides Obtains correct equation for $\sin^2\theta$ Sqrts (allow omission of ±), finds the principal value and uses a correct method to find at least one other angle in range At least two angles correct All four angles correct
5 (c)	• so that $\log(5 - 3\sin^2\theta)$ is defined • $\log x$ is not defined for $x \le 0$ • make sure all solutions are valid	B1 **[1]**	A correct explanation/illustration of why (a) is useful

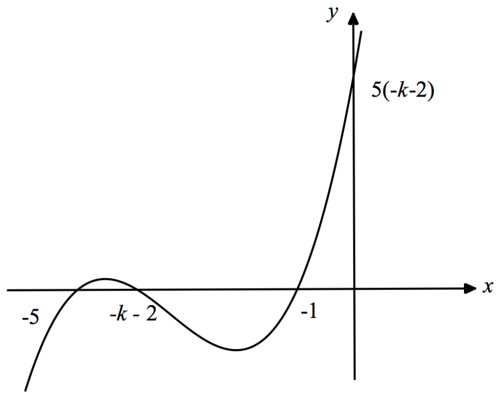
6	$5000 = \lim_{t\to\infty} \frac{A}{1+B\mathrm{e}^{-Ct}} = A$, **so *A* = 5000**	B1	Correct value of *A*
	$50 = \frac{5000}{1+B\mathrm{e}^{-C(0)}}$ $\Rightarrow 50 = \frac{5000}{1+B}$	M1	Substitutes $t = 0$ and $N = 50$ into equation to form an equation in *B* Allow equation to be in terms of *A* for this mark
	$\Rightarrow$ ***B* = 99**	A1	Correct value of *B*
	$150 = \frac{5000}{1+99\mathrm{e}^{-C}} \Rightarrow \mathrm{e}^{-C} = \ldots$ $\Rightarrow \mathrm{e}^{-C} = 0.3265\ldots$	M1*	Substitutes $N = 150$ at $t = 1$ (must have some values for *A* and *B* at this stage) and rearranges for e^{-C}
	$\Rightarrow -C = \ln 0.3265\ldots$ $\Rightarrow C = -\ln 0.3265\ldots$	M1(dep*)	Takes natural logs with use of ln(e) = 1 seen
	So ***C* = 1.11(90…)**	A1 **[6]**	Correct value of *C*. Awrt 1.12

7 (a)	$\left(3-\frac{x}{5}\right)^5 = 3^5 + \binom{5}{1}(3)^4\left(-\frac{x}{5}\right) + \binom{5}{2}(3)^3\left(-\frac{x}{5}\right)^2 + \ldots$ $= 243 - 81x + \frac{54}{5}x^2 - \ldots$ $\left(2+\frac{x}{3}\right)\left(3-\frac{x}{5}\right) = \left(2+\frac{x}{3}\right)\left(243-81x+\frac{54}{5}x^2+\ldots\right)$ $= 486 - 162x + \frac{108}{5}x^2 + 81x - \frac{81}{3}x^2 + \ldots$ $= 486 - 81x - \frac{27}{5}x^2 + \ldots$	M1* A1 M1(dep*) A1 **[4]**	Award for at least one term of the form ${}^5C_k\,(3)^{5-k}(-x/5)^k$, $0 < k < 5$ Correct unsimplified expansion up to x^2 Attempts to find the full expansion by multiplying out the brackets to obtain expression with correct number of terms Correct expansion up to x^2
7 (b)	${}^nC_2 = \frac{n!}{2!(n-2)!}$ $= \frac{1}{2} \times \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$ $= \frac{1}{2}n(n-1)$ (as required)	M1* A1 **[2]**	Writes down correct definition of ${}^nC_2$ Shows the result convincingly
7 (c)	${}^pC_2(5)^2(1)^{p-2} = 700$ $\Rightarrow \frac{25}{2}p(p-1) = 700$ $\Rightarrow p^2 - p = 56$ $\Rightarrow p^2 - p - 56 = 0$ $\Rightarrow (p-8)(p+7) = 0$ And $p > 0$, so $p = 8$	M1 A1 **[2]**	Forms correct equation in terms of p and attempts to solve for p Obtains correct value of p

8	Let $y = x^2 - 2x^3$ $\frac{dy}{dx} = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h}$ $= \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h)^3 - (x^2 - 2x^3)}{h}$ $= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 2(x^3 + 3x^2h + 3xh^2 + h^3) - x^2 + 2x^3}{h}$ $= \lim_{h \to 0} \frac{2xh + h^2 - 6x^2h - 6xh^2 - 2h^3}{h}$ $= \lim_{h \to 0}(2x - 6x^2 + h - 6xh - 2h^2)$ $= 2x - 6x^2$ Then $\frac{d^2y}{dx^2} = \lim_{h \to 0} \frac{y'(x+h) - y'(x)}{h}$ $= \lim_{h \to 0} \frac{2(x+h) - 6(x+h)^2 - 2x + 6x^2}{h}$ $= \lim_{h \to 0} \frac{2x + 2h - 6x^2 - 12xh - 6h^2 - 2x + 6x^2}{h}$ $= \lim_{h \to 0} \frac{2h - 12xh - 6h^2}{h}$ $= \lim_{h \to 0}(2 - 12x + h - 6h)$ $= 2 - 12x$ **AG**	M1* M1 A1 M1(dep*) A1 cso **[5]**	Writes down correct limit definition of the first derivative Attempts to expand all brackets and collect-like terms in the numerator (NOTE: this can be awarded before the 1st M1 in the scheme, as candidates may not divide by *h* until later) Obtains correct first derivative with correct limiting process seen Complete method to find the second derivative using correct limit definition (ft their first derivative) Shows the result convincingly with correct limiting process seen and no errors
9 (a)	$2x - 4y - 10 = 0 \Rightarrow y = \frac{1}{2}x - \frac{5}{2}$ So gradient of $l_1 = 1/2$ Hence gradient of $l_2 = -2$	B1 B1ft **[2]**	Obtains correct gradient of l_1 Correct gradient of l_2 ft their gradient of l_1 **Answer only is 2/2**

9 (b)	Equation of l_2 is $y = -2x + k$ So crosses y axis at $(0, k)$ and x axis at $(k/2, 0)$ $4 = \frac{1}{2}(k)\left(\frac{k}{2}\right) \Rightarrow k^2 = 16$ $\Rightarrow k = \pm 4$ However can see from the diagram that $k < 0$, so $k = -4$ Hence **<u>coordinates of A = (–2, 0)</u>** and **<u>coordinates of B = (0, –4)</u>**	M1* M1(dep*) A1 A1 **[4]**	Characterises the points A and B in some way in terms of one variable Uses their characterisation to write down an equation in terms of their variable and attempts to find its value Correct value of their k (allow +4 (or both) for this mark) Obtains the correct coordinates
9 (c)	Equation of the line is then $y = -2x - 4$ $\Rightarrow$ **<u>$2x + y + 4 = 0$</u>**	B1 **[1]**	Correct equation the line in the required form (allow non-zero integer multiples)
9 (d)	At intersection point C: $2x - 4(-2x - 4) - 10 = 0$ $\Rightarrow 2x + 8x + 16 - 10 = 0$ $\Rightarrow x = -\frac{3}{5}$ So $y = -\frac{14}{5}$ Coordinates of $D = \left(0, -\frac{5}{2}\right)$ Area of quad. $OACD$ = area of tr. OAB – area of tr. BCD Area of $BCD = \frac{1}{2}\left(\frac{3}{5}\right)\left(4 - \frac{5}{2}\right) = \frac{9}{20}$ Hence area of $OACD = 4 - \frac{9}{20} = \frac{71}{20}$	M1 A1 B1 M1 A1 **[5]**	Uses a correct method to find the coordinates of C Correct coordinates of C Correct coordinates of D stated or implied Complete method to find the area of the quadrilateral ALT: may split $OACD$ into a right-angled triangle and a trapezium. M1 for correct method to find their individual areas and combining Correct area of the quadrilateral

10 (a)	$a = 26 - b$ cosine rule gives $\cos\theta = \dfrac{14^2 + b^2 - a^2}{2(14)b}$ $\Rightarrow \cos\theta = \dfrac{14^2 + b^2 - (26-b)^2}{28b}$ $\Rightarrow \cos\theta = \dfrac{196 + b^2 - (676 - 52b + b^2)}{28b}$ $\Rightarrow \cos\theta = \dfrac{-480 + 52b}{28b}$ $\Rightarrow \cos\theta = \dfrac{13}{7} - \dfrac{120}{7}b$ **AG**	B1 M1* M1(dep*) A1 **[4]**	Correct a in terms of b seen or implied Uses the cosine rule to form an expression for $\cos\theta$ in terms of a and b Substitutes a in their cosine rule expression with their $26 - b$ Complete and convincing proof with no errors seen
10 (b)	$A^2 = \dfrac{1}{2}b^2(14)^2\sin^2\theta$ $= 49b^2(1 - \cos^2\theta)$ $= 49b^2\left(1 - \left(\dfrac{13}{7} - \dfrac{120}{7b}\right)^2\right)$ $= 49b^2\left(-\dfrac{120}{49} + \dfrac{3120}{49b} - \dfrac{14400}{49b^2}\right)$ $= -120b^2 + 3120b - 14400$ **AG**	M1* M1**(dep*) M1(dep**) A1 **[4]**	Writes down correct expression for A^2 Uses $\sin^2\theta = 1 - \cos^2\theta$ to write expression for A^2 in terms of $\cos\theta$ Uses part (a) to obtain an expression for A^2 in terms of b Obtains given result convincingly with no errors seen
10 (c) (i)	$A^2 = -120(b^2 - 26b + 120)$ $= -120\left[(b-13)^2 - 13^2 + 120\right]$ $= -120(b-13)^2 + 5880$ So $\max(A^2) = 5880 \Rightarrow \max(A) =$ awrt 76.7	M1 M1 A1 B1ft **[4]**	Extracts a factor of –120 from expression (or first two terms) and attempts to complete the square Obtains $-120(b - 13)^2 + \ldots$ Completes the square correctly Correct maximum value of A ft their completing the square (provided their $5880 > 0$)

10 (c) (i) ALT	$\frac{d(A^2)}{db} = -240b + 3120$ Max. occurs when $-240b + 3120 = 0 \Rightarrow b = 13$ So $\max(A^2) = -120(13)^2 + 3120(13) - 14400 = 5880$ $\Rightarrow \max(A) =$ awrt 76.7	M1* A1 M1(dep*) A1ft **[4]**	Correctly differentiates the expression and sets it equal to 0 Correct value of b at maximum Substitutes their b at the maximum into A^2 and uses it to find maximum of A Correct maximum of A ft their b **Note the scheme change here, final mark is A1ft <u>not B1ft</u>**
10 (c) (ii)	Max happens when $b = 13 \Rightarrow a = 13$, so the triangle is <u>**isosceles**</u>	B1 **[1]**	Correct type of triangle stated (not reason needed)
11 (a)		B1 B1 B1 **[3]**	Correct shape of the graph Correct x intersections labelled Correct y intersection labelled

11 (b)	$(x-1)(x+3)(x+k)=(x^2+2x-3)(x+k)$ $=x^3+(k+2)x^2+(2k-3)x-3k$	M1[1]	Attempts to expands brackets obtaining cubic expression in *k* with correct no. of terms
	$\int_0^1 y\,dx=\int_0^1(x^3+(k+2)x^2+(2k-3)x-3k)\,dx$	M1[2](dep1)	Integrates their cubic expression indefinitely
	$=\left[\frac{x^4}{4}+\frac{k+2}{3}x^3+\frac{2k-3}{2}x^2-3kx\right]_0^1$	A1	Correct indefinite integration
	$=\frac{1}{4}+\frac{k+2}{3}+\frac{2k-3}{2}-3k$ $=\frac{3+4k+8+12k-18-36k}{12}$	M1[3](dep2)	Substitutes correct limits into their indefinite integral in the correct order. NB: allow use of integration from 1 to 0, but they must be consistent with the order of limits on the integral and the order of substitution
	$=\frac{-7-20k}{12}$ So area between 0 and 1 is $\frac{7+20k}{12}$	A1	Correct unsimplified area of the curve between 0 and 1 **<u>(up to sign)</u>**
	Then area of triangle *OAB* is $\frac{1}{2}(k)(3k)=\frac{3k^2}{2}$	B1	Correct area of the triangle *OAB*
	Hence $\frac{3k^2}{2}+\frac{7+20k}{12}=\frac{119}{12}$	M1(dep3)	Combines their two areas to obtain a quadratic in *k* and attempts to express in the form of a 3TQ
	$\Rightarrow 18k^2+7+20k-119=0$ $\Rightarrow 18k^2+20k-112=0$ $\Rightarrow 9k^2+10k-56=0$ **AG**	A1 cso **[7]**	Shows the result convincingly with no errors
11 (c)	(Using a calculator and the figure means that) <u>$k=2$</u>	B1 **[1]**	Correct value of *k* (no other values allowed)

12 (a)	Model octopus as $(x+7)^2+(y-5)^2=25$ Fish's path intersects circular region if $(x+7)^2+(kx+1)^2=25$ $\Rightarrow x^2+14x+49+k^2x^2+2kx+1=25$ $\Rightarrow (1+k^2)x^2+(14+2k)x+25=0$ Octopus does not catch the fish if $(14+2k)^2-4(1+k^2)(25)<0$ $\Rightarrow\ 196+56k+4k^2-100-100k^2<0$ $\Rightarrow\ -96k^2+56k+96<0$ $\Rightarrow\ 12k^2-7k-12>0$ (after dividing by –8) For CVs, consider $12k^2-7k-12=0$ $\Rightarrow (3x-4)(4x+3)>0$ $\Rightarrow x=\frac{4}{3},-\frac{3}{4}$ So values of k satisfying inequality are $k<-3/4$ or $k>4/3$ As a set, therefore we have $\{k \in \mathrm{R} : k<-3/4 \text{ or } k>4/3\}$	B1 B1 M1* A1 oe M1**(dep*) A1 M1(dep**) A1 **[8]**	Correct LHS of circle Correct RHS of circle Eliminates y from their equation of the circle and attempts to expand and form a 3TQ (or eliminates x) Correct 3TQ in x ***OR*: correct 3TQ in y which is** $(1+k^2)y^2+(14k-12-10k^2)y+(36-84k+49k^2)=0$ Correct condition for equation to have no solutions Alt: can complete the square and set '+ constant term' > 0 Correct inequality for k Complete method to find CVs of their quadratic inequality in k Obtains correct set of integer values of k Allow $\{k: k<-3/4 \text{ or } k>4/3\}$ or $\{k: k<-3/4\} \cup \{k: k>4/3\}$
12 (b)	Any one from: • fish may not swim in straight line • fish may not swim on a straight line with gradient 4 • size of fish not considered • octopus may move away from the origin • size of octopus not considered	B1 **[1]**	One correct limitation – mark to the advantage of the candidate if more than limitation given