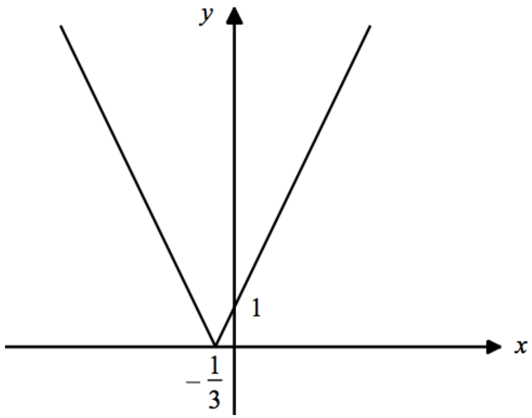
A Level
Maths

Bronze Set B, Paper 2 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1 (a)	$(4-\sqrt{3})^2 = 16 - 4\sqrt{3} - 4\sqrt{3} + 3$ $= 19 - 8\sqrt{3}$ so $a = 19$, $b = -8$	M1 A1 **[2]**	Expands brackets obtaining correct number of terms. Only condone sign errors Correct answer in the correct form or a and b stated Answer only is 2/2
1 (b)	$\frac{3-\sqrt{3}}{2+\sqrt{3}} = \frac{(3-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$ $= \frac{6-3\sqrt{3}-2\sqrt{3}+3}{4-3}$ $= 9-5\sqrt{3}$	M1* M1(dep*) A1 A1 **[4]**	Multiplies top and bottom by $2 - \sqrt{3}$ Attempts to expand numerator obtaining correct number of terms. You are only condoning sign errors Correct unsimplified numerator or denominator Correct answer in the correct form M1 M0 A1 A0 is possible
2 (a)	$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ p \end{pmatrix} = 4\mathbf{i} + 3\mathbf{j} + (-1-p)\mathbf{k}$	M1 A1 **[2]**	Attempts to find AB (give the M1 if they find the vector BA) Obtains the correct answer
2 (b)	$5\sqrt{2} = \sqrt{4^2 + 3^2 + (-1-p)^2}$ $\Rightarrow 50 = 16 + 9 + p^2 + 2p + 1$ $\Rightarrow p^2 + 2p - 24 = 0$ $\Rightarrow (p+6)(p-4) = 0$ and since $p > 0$, $p = 4$	M1* A1 M1(dep*) A1 **[4]**	Forms a correct equation using their (a) Obtains the correct 3TQ Complete method to solve their 3TQ Correct value of p A0 if both solutions to the quadratic given

3 (a)	$f(x) = 2 - \frac{5}{x+5} + \frac{5}{(x+5)^2}$ $= \frac{2(x+5)^2}{(x+5)^2} - \frac{5(x+5)}{(x+5)^2} + \frac{5}{(x+5)^2}$ $= \frac{2(x+5)^2 - 5(x+5) + 5}{(x+5)^2}$ $= \frac{2x^2 + 20x + 50 - 5x - 25 + 5}{(x+5)^2}$ $= \frac{2x^2 + 15x + 30}{(x+5)^2}$	M1 A1 A1 **[3]**	Attempts to make denominators of at least two fractions the same Combines the fractions correctly Obtains the given result convincingly with no errors seen
3 (b)	**R1 (completing the square):** $2x^2 + 15x + 30 = 2\left(x^2 + \frac{15}{2}\right) + 30$ $= 2\left[\left(x + \frac{15}{4}\right)^2 - \frac{225}{16}\right] + 30$ $= 2\left(x + \frac{15}{4}\right)^2 + \frac{15}{8}$ $\left(x + \frac{15}{4}\right)^2 \geq 0$ for all x, so $2\left(x + \frac{15}{4}\right)^2 + \frac{15}{8} > 0$ for all x	M1 A1 A1 **[3]**	Attempts to complete the square Completes the square correctly + gives a comment, i.e. $\left(x + \frac{15}{4}\right)^2 \geq 0$ or coordinates of min. point. Condone $\left(x + \frac{15}{4}\right)^2 > 0$ for this mark Complete and convincing proof with no errors seen
3 (b) ALT	**R2 (discriminant)**: $15^2 - 4(2)(30) = -15$ Discriminant is –15 < 0 which means the there are no real roots And since the graph of $2x^2 + 15x + 30$ is U-shaped, it follows that $2x^2 + 15x + 30 > 0$ for all x	M1 A1 A1 **[3]**	Attempts to calculate the discriminant Correct value of the discriminant **and** states that there are no real roots ***OR*** that the graph is U-shaped (equivalently, they can say that the coefficient of x^2 is 2 > 0) Complete and convincing proof with no errors seen. Must see correct value of discriminant, mention of no real roots and that the graph is U-shaped

3 (b) ALT	**R3 (differentiation)**: Derivative is $4x + 15$, so turning point when $4x + 15 = 0$ This gives $x = -15/4$, which gives $y = 15/8$ Since the graph is U-shaped, this is a minimum point and so $2x^2 + 15x + 30 > 0$ for all x	 M1 A1 A1 **[3]**	 Complete method to find coordinates of the turning point Correct coordinates of turning point States graph is U-shaped or uses second derivative to show the point is a minimum to give a complete and convincing proof. No errors seen
3 (c)	Numerator from (b) is positive $x \neq -5 \Rightarrow (x+5)^2 > 0$ (denominator is positive) Hence $f(x) > 0$ for all x, $x \neq 5$	B1 **[1]**	Convincingly shows $f(x) > 0$
4 (a)		B1 B1 B1 **[3]**	Correct shape **including cusp** (a smooth turning point is B0) Correct x intersection Correct y intersection
4 (b)	$3x+1 = 3-x$ or $-(3x+1) = 3-x$ $\Rightarrow 4x = 2$ or $\Rightarrow -3x-1 = 3-x$ $\Rightarrow x = \frac{1}{2}$; $\Rightarrow x = -2$	M1 A1 A1 **[3]**	States or implies the two correct equations not involving moduli One value of x correct Both values of x correct **SC:** a single correct value of x scores SCB1 M0 A0

4 (b) ALT	$(3x+1)^2 = (3-x)^2$ $\Rightarrow 9x^2 + 6x + 1 = 9 - 6x + x^2$ $\Rightarrow 2x^2 + 3x - 2 = 0$ $\Rightarrow (2x-1)(x+2) = 0$ so $x = \frac{1}{2}$ or $x = -2$	M1 A1 A1 **[3]**	Squares both sides and attempts to solve the equation for x One value of x correct Both values of x correct
5	$2y^2 - 7y + 3 = 0$ $\Rightarrow (2y-1)(y-3) = 0$ $\Rightarrow y = \frac{1}{2}$ or $y = 3$ Then $2^x = \frac{1}{2}$ or $2^x = 3$ $\Rightarrow x = -1$ or $x = \log_2 3 = 1.584...$	M1 A1 A1 A1 **[4]**	Writes down correct equation in terms of y Correct values of y $(2(2^x) - 1)(2^x - 3) = 0$ is M1 A1 One correct value of x A second correct value of x
6	$\ln y = x \ln(\sin x)$ Differentiating both sides wrt x gives $\frac{1}{y}\frac{dy}{dx} = \ln(\sin x) + x\left(\frac{\cos x}{\sin x}\right)$ $\Rightarrow \frac{dy}{dx} = y\left(\ln(\sin x) + x\cot x\right)$ $= (x\cot x + \ln(\sin x))(\sin x)^x$ **AG**	M1* M1(dep*) M1(dep*) A1 A1 **[5]**	Takes logs to both sides and uses the power rule Implicitly differentiates LHS to obtain terms y^{-1} and dy/dx Attempts to use the product rule on the RHS Correct differentiation (unsimplified or better) Obtains the given result convincingly with no errors seen

7 (a) (i)	Centre of the circle is at the midpoint of *AB* which is $\left(\frac{4+8}{2},\frac{5+2}{2}\right)=\left(6,\frac{7}{2}\right)$ **AG** Radius of the circle is $\frac{1}{2}\sqrt{(8-4)^2+(2-5)^2}=\frac{1}{2}\sqrt{4^2+(-3)^2}=\frac{5}{2}$ **AG**	B1 M1 A1 **[3]**	Works out the midpoint of *AB* to obtain the given coordinates of the centre of the circle For sight of $\sqrt{[(8-4)^2+(2-5)^2]}$ or equivalent Obtains the radius of the circle convincingly
7 (a) (ii)	$(x-6)^2+\left(y-\frac{7}{2}\right)^2=\frac{25}{4}$	B1 B1 **[2]**	LHS correct RHS correct
7 (b)	$PM=\sqrt{(6-5)^2+\left(\frac{7}{2}-9\right)^2}=\frac{5\sqrt{5}}{2}$ (Angle *PTM* is right-angled, so using right-angle trig gives:) $\cos TMQ=\frac{2.5}{\left(\frac{5\sqrt{5}}{2}\right)}\Rightarrow TMQ=1.10714....$ so *TMQ* is 1.1071 to 4 dp as required **AG**	B1 M1 A1 **[3]**	Correct length of *PM* seen or implied Complete method to find the angle *TMQ* ft their *PM* Obtains the given result convincingly with no errors

7 (c)	Area of the triangle is $=\frac{1}{2}(PM)(TM)\sin TMQ$ $=\frac{1}{2}\left(\frac{5\sqrt{5}}{2}\right)(2.5)\sin(1.10714...)$ $=\frac{25}{4}$ Area of sector is $\frac{1}{2}(TMQ)r^2=\frac{1}{2}(1.10714...)(2.5)^2=3.459...$ So area of the shaded region is $\frac{25}{4}-3.459...=2.79...$ units^2	M1 A1 B1 M1 A1 **[5]**	Complete method to find the area of the triangle Alternatively, you may see candidates use Pythagoras to find *PT* and then use "½ × base × height" to find the area Correct area of the triangle Correct area of the sector seen or implied Area of shaded region = their area of triangle – their area of sector Correct area of the shaded region
8 (a)	$\int_1^p (\ln x+3)\,dx=12$ $\int \ln x\,dx = x\ln x - x$ so $[x\ln x - x + 3x]_1^p = 12$ $\Rightarrow p\ln p - p + 3p - (1\ln 1 - 1 + 3(1)) = 12$ $\Rightarrow p\ln p + 2p + 1 - 3 - 12 = 0$ $\Rightarrow p\ln p + 2p - 14 = 0$ **AG**	M1 M1 A1 M1 A1 **[5]**	Forming a correct **equation** using the given information at any stage and in any form E.g. if they find integral, sub in limits, then equate, the M1 is awarded when they equate their area = 12 Method to find the integral of ln(*x*), i.e. using parts the correct way Correct integral of ln(*x*) Both these marks can be implied Substitutes in limits in the correct order into **their** expression for the integral Obtains the correct answer convincingly with no errors seen

8 (b)	$p_1 = \frac{14}{2+\ln(4)} = 4.1343127$ $\Rightarrow p_2 = 4.0943800$ $\Rightarrow p_3 = 4.1060350$ $\Rightarrow p_4 = 4.1026147$ $\Rightarrow p_5 = 4.1036168$ so $\alpha = 4.10$	M1 A1 A1 **[3]**	Uses the iteration formula correctly once Obtains $\alpha = 4.10$ Shows sufficient iterations to 4 dp to justify $\alpha = 4.10$ (need to see two consecutive iterations which round to 4.10) Do not penalise values of p given to more than 4 dp Do not penalise small slips in giving values of p to 4 dp **NB: values in mark scheme are truncated at 7 dp**
9 (a)	$\frac{3\cos x}{\sin x} + 1 = \frac{2}{\sin x} \Rightarrow 3\cos x + \sin x = 2$, so $a = 3$, $b = 1$, $c = 2$	M1 A1 **[2]**	Writes cot and cosec correctly in terms of sin and cos Obtains the correct equation convincingly Values of a, b and c do not need to be stated
9 (b)	Let $3\cos x + \sin x = R\cos(x+\alpha)$, then $3\cos x + \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$ So $R\cos\alpha = 3$, $R\sin\alpha = -1$ $\Rightarrow R = \sqrt{10}$ and $\alpha = \tan^{-1}\left(-\frac{1}{3}\right) = -18.434...$ So $3\cos x + \sin x = \sqrt{10}\cos(x - 18.434...)$ $\sqrt{10}\cos(x - 18.434...) = 2$ $\Rightarrow x - 18.434... = 50.768...$ Other value in range is 360 – 50.768… = 309.231… So values of x are 69.2 and 327.7	M1* M1(dep*) A1 M1(dep*) A1 A1 **[6]**	Shows intention to express 3cos$x$ + sin$x$ in the form Rcos(x + α) Complete method to find the values of R and α (you are only condoning sign errors) Correct expression Uses their Rcos(x + α) form and finds the principal value of the argument One value of x correct to at least 1 dp Second value of x correct to at least 1 dp

9 (b) ALT	Let $3\cos x + \sin x = R\sin(x+\alpha)$, then $3\cos x + \sin x = R\sin x\cos\alpha + R\cos x\sin\alpha$ So $R\cos\alpha = 1,\ R\sin\alpha = 3$ $\Rightarrow R = \sqrt{10}$ and $\alpha = \tan^{-1}(3) = 71.565...$ So $3\cos x + \sin x = \sqrt{10}\sin(x + 71.565...)$ $\sqrt{10}\sin(x+71.565...) = 2$ $\Rightarrow x + 71.565... = 39.231...$ (which is not in range) Values in range is 180 + 71.565… = 140.768… and 360 + 76.565… = 399.231… So values of x are 69.2 and 327.7	M1* M1(dep*) A1 M1(dep*) A1 A1 **[6]**	Shows intention to express 3cos$x$ + sin$x$ in the form Rsin(x + α) Complete method to find the values of R and α (you are only condoning sign errors) Correct expression Uses their Rcos(x + α) form and finds the principal value of the argument One value of x correct to at least 1 dp Second value of x correct to at least 1 dp
9 (b) ALT	**Use of t = tan(x/2) (this is a used in Further Pure 2):** $3\cos x + \sin x = 2$ $\Rightarrow 3\left(\frac{1-t^2}{1+t^2}\right) + \frac{2t}{1+t^2} = 2$ $\Rightarrow 3 - 3t^2 + 2t = 2 + 2t^2$ $\Rightarrow 5t^2 - 2t - 1 = 0$ $\Rightarrow t = \frac{1 \pm \sqrt{6}}{5}$ Then $\tan\left(\frac{x}{2}\right) = \frac{1+\sqrt{6}}{5}$ or $\tan\left(\frac{x}{2}\right) = \frac{1-\sqrt{6}}{5}$ $\Rightarrow \frac{x}{2} = 34.601... + 180n$ or $\frac{x}{2} = -16.166... + 180n$ Which gives values of x in range to be 69.2 and 327.7	M1* M1**(dep*) A1 M1(dep**) A1 A1 **[6]**	Substitutes **correct** expressions for cos$x$ and sin$x$ in terms of t Attempts to re-arrange expression to form a 3TQ Obtains the correct 3TQ Complete method to solve their 3TQ and use their solutions to find the two principal values of x/2 One value of x correct to at least 1 dp Second value of x correct to at least 1 dp

10 (a)	$N = 120e^0 + e^0 = 121$	B1 **[1]**	Cao
10 (b)	$\frac{dN}{dt} = 120(-0.9)e^{-0.9t} + 2e^{2t}$ $= -108e^{-0.9t} + 2e^{2t}$ For decreasing N, we need $\frac{dN}{dt} < 0 \Rightarrow -108e^{-0.9t} + 2e^{2t} < 0$ $\Rightarrow e^{2t} < 54e^{-0.9t}$ $\Rightarrow e^{2.9t} < 54$ $\Rightarrow 2.9t < \ln 54$ $\Rightarrow t < 1.3755...$ so t is decreasing for $(0 \le)\ t < 1.3755\ldots$	M1* A1 M1**(dep*) M1(dep**) A1 **[5]**	Attempts to find the derivative Correct derivative Sets their derivative < 0 Employs a complete method to find the set of values of t for which their derivative < 0 Correct values of t Condone omission of the lower tail in the range of time

10 (c)	$N \to \infty$ as $t \to \infty$	B1 **[1]**	Correct long term behaviour
10 (d)	The population survives the disease outbreak since, ***then any one from the following*** • N increases after $t = 1.3755$ / the population decreases but then starts to increase again (without limit) • $N \to \infty$ as $t \to \infty$ • N is never 0 / always positive	B1 **[1]**	States that the population survives the outbreak and gives a supporting reason from the list (accept equivalent formulations of these reasons)
10 (e)	The model predicts the population will grow arbitrarily/infinitely large	B1 **[1]**	Limitation
11 (a)	$\frac{dy}{dx} = \frac{dy}{ds}\frac{ds}{dx} = 2\left(\frac{1}{2s}\right) = \frac{1}{s}$ so at P, gradient is $\frac{1}{p}$ and the normal has gradient $-p$ Hence equation of normal is $y - 2p = -p(x - p^2)$ $\Rightarrow y - 2p = -px + p^3$ $\Rightarrow y + px = p^3 + 2p$ **AG**	M1* A1 A1ft M1(dep*) A1 **[5]**	Method to find dy/dx Correct expression for dy/dx in terms of s **OR** x and y Correct gradient of the normal at P ft their dy/dx Attempts to find the normal using their gradient and correct expressions for the x and y coordinate Obtains the given result convincingly with no errors
11 (a) ALT	**Alternative method for calculating $\frac{dy}{dx}$:** Finds equation of the curve: $x = \left(\frac{y}{2}\right)^2 \Rightarrow y^2 = 4x$ So $2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y} \quad \left(= \frac{1}{s}\right)$	 M1	**This only concerns the 1st M1 in the main scheme** M1 awarded for correct equation of the curve (condone an error with squaring the factor of 2) and attempting to use implicit differentiation

11 (b) (i)	The point Q has coordinates $(15, -12)$ Substituting this into the normal gives $-12 + p(15) = p^3 + 2p \Rightarrow p^3 - 13p + 12 = 0$ **AG**	M1 A1 **[2]**	Attempts to find coordinates of Q **and** substitutes their coordinates into the normal line Obtains the given answer convincingly with no errors seen
11 (b) (ii)	$3^3 - 13(3) + 12 = 27 - 39 + 12 = 0$, so (by the factor theorem,) p could be 3 $p^3 - 13p + 12 = (p-3)(p^2 + 3p - 4)$ so p could also be –4 and 1 Hence possible coordinates of P are $(9, 6)$, $(16, -8)$ and $(1, 2)$	B1 M1 B1 A1 **[4]**	Shows that p could be 3 convincingly Complete method to find the other values of p (e.g. use of long division/inspection to find quadratic factor **and** attempting to solve that quadratic) For the coordinate (9, 6) For the other two coordinates: (16, –8) and (1, 2)
12 (a)	$S_n = a + ar + ar^2 + ... + ar^{n-1}$ $rS_n = ar + ar^2 + ar^3 + ... + ar^n$ $S_n - rS_n = a - ar^n$ $\Rightarrow S_n(1-r) = a(1-r^n)$ $\Rightarrow S_n = \frac{a(1-r^n)}{1-r}$ **AG**	M1 M1 A1 **[3]**	Writes out the series for S_n with at least three terms, including first and last, and multiplies the series by r $S_n = \ldots$ and $rS_n = \ldots$ not required Subtracts the series Obtains the given result convincingly with no errors **SC:** if they work out the sum of the first $(n + 1)$ terms, award at most M0 M1 A0
12 (b)	$\sum_{k=1}^{8} 10(2^k) = \frac{20(1-2^8)}{1-2} = 5100$	M1 A1 **[2]**	Identifies $a = 20$ and $r = 2$ Correct sum

12 (c)	$\lvert r \rvert < 1$	B1 **[1]**	Correct condition Allow 'magnitude of common ratio is less than 1' / 'common ratio is between –1 and 1' oe Do not allow 'common ratio less than 1'
12 (d)	$(\frac{8}{9} \div \frac{4}{3} = \frac{2}{3}$, so) common ratio is $\frac{2}{3}$, and $\left\lvert \frac{2}{3} \right\rvert < 1$, so the series converges Hence $\frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \ldots = \frac{\frac{4}{3}}{1 - \frac{2}{3}} = 4$	B1 M1 A1 **[3]**	Finds the common ratio and explains why the series converges If they've stated the condition correctly in (c), allow a comment such as 'common ratio is 2/3 which satisfies the condition in (c)' Applies the infinite geometric series formula Correct value of the series
13 (a)	$4y + 2x - 10 = 0 \Rightarrow y = -\frac{1}{2}x + \frac{5}{2}$ so gradient of l is 2	M1 A1 **[2]**	Attempts to re-arrange the equation of the line into the form $y = mx + c$ Correct gradient of l
13 (b)	$2x + 4 = 0 \Rightarrow x = -2$, so coordinates are $(-2, 0)$	B1 **[1]**	Correct coordinates (allow $x = -2$, $y = 0$) Allow for $x = -2$ alone
13 (c)	$6 = 2x + 4 \Rightarrow x = 1$, so coordinates of Q are $(1, 6)$	B1 **[1]**	Correct coordinates (allow $x = 1$, $y = 6$) $x = 1$ alone is B0
13 (d)	Area of trapezium $PQRS = \left(\frac{(a+2)+(a-1)}{2}\right)(6) = 3(2a+1)$ Area is $7a \Rightarrow 3(2a+1) = 7a$ $\Rightarrow 2a + 1 = 7a/3$ $\Rightarrow a = 3$	M1* A1 M1(dep*) A1 **[4]**	Complete method to find the area of the shape If they split it up, must attempt to find the area of each region Correct expression for the area Sets their expression for the area = $7a$ and solves for a Obtains the correct value of a