Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				

Examiner Comments		Total Marks

FURTHER MATHEMATICS

A LEVEL CORE PURE 2

CM

Bronze Set B (Edexcel Version)

Time allowed: 1 hour and 30 minutes

Instructions to candidates:

• In the boxes above, write your centre number, candidate number, your surname, other names and signature.

• Answer ALL of the questions.

• You must write your answer for each question in the spaces provided.

• You may use a calculator.

Information to candidates:

• Full marks may only be obtained for answers to ALL of the questions.

• The marks for individual questions and parts of the questions are shown in round brackets.

• There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to candidates:

• You should ensure your answers to parts of the question are clearly labelled.

• You should show sufficient working to make your workings clear to the Examiner.

• Answers without working may not gain full credit.

1 1 3 3 1 2 2 2 8 0 0 0 4

1 A computer company sells three types of laptops to three nearby stores. The table below shows the quantity of each laptop ordered by each store and the total cost of their purchase.

Store	**Laptop** A	**Laptop** B	**Laptop** C	**Total cost** (£)
Store A	13	9	10	17 080
Store B	8	1	6	8 920
Store C	11	10	15	21 100

Using the information in the table, form and solve a matrix equation to find the price of each laptop. Show your workings clearly. **(4)**

Question 1 continued

TOTAL 4 MARKS

2 The cubic equation

$$5x^3 - x^2 + 2x + 1 = 0$$

has roots α, β and γ.

Without solving the equation, find a cubic equation, with integer coefficients, that has roots $(\alpha + 4)$, $(\beta + 4)$ and $(\gamma + 4)$. **(4)**

Question 2 continued

TOTAL 4 MARKS

3 The curve C has polar equation

$$r = 3 + 2\sin\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

The point P lies on C such that the tangent to C at P is perpendicular to the initial line.

Given that O is the pole, find the exact length OP. **(7)**

Question 3 continued

TOTAL 7 MARKS

4 The line l_1 passes through the points A and B has the equation $\mathbf{r} = (\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(4\mathbf{i} - \mathbf{j} - \mathbf{k})$, where t is a scalar parameter.

The line l_2 has the equation $\dfrac{x-4}{2} = \dfrac{1-y}{3} = \dfrac{z}{5}$.

Determine whether the lines l_1 and l_2 intersect, are skew or neither.
Show all of your working. **(5)**

Question 4 continued

TOTAL 5 MARKS

5 (a) By starting with the exponential definition of $\cosh x$, show that

$$\operatorname{arcosh} x = \ln\left[x + \sqrt{x^2 - 1}\right], \quad x \geq 1$$

(4)

(b) **Hence**, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2 - 1}}$$

(3)

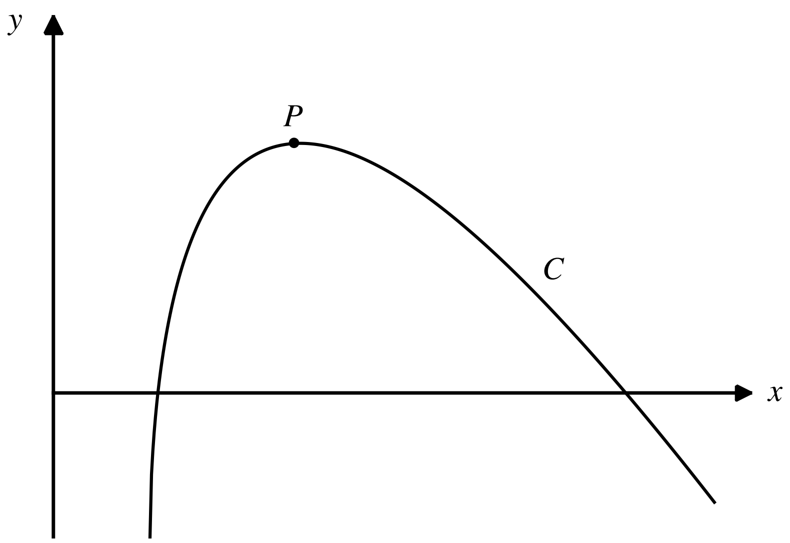

Figure 1

Figure 1 above shows a sketch of the curve C with equation $y = 12\operatorname{arcosh} x - 5x$, $x \geq 1$. The curve C has a maximum point P.

Given that the coordinates of P are $(a, b\ln 5 + c)$,

(c) find the values of the constants a, b and c.

(5)

Question 5 continued

Do not write outside the box

Question 5 continued

Question 5 continued

TOTAL 12 MARKS

6 The matrix **P** is defined such that

$$\mathbf{P} = \begin{pmatrix} 7 & 24 \\ 24 & -7 \end{pmatrix}$$

The matrix **P** represents a linear transformation, T, of the plane.

(a) Show that the origin is the only point invariant under T. **(2)**

(b) Describe the invariant lines of the transformation T. **(6)**

Question 6 continued

TOTAL 8 MARKS

7 The complex number $u = 3 + 3\text{i}$.

Given that u solves the quadratic $x^2 + ax + b = 0$,

(a) find the values of a and b. **(3)**

(b) Show the complex numbers 1, i and u on an Argand diagram. **(1)**

(c) On the **same** Argand diagram as (b), shade the region that is included in the set

$$X = \{z \in \mathbb{C} : |z - 1| \leq |z - \text{i}|\} \cap \{z \in \mathbb{C} : |z - u| \leq 1\}$$

(3)

(d) Using your diagram, calculate the value of $|z|$ for the point in X for which $\arg z$ is least. **(3)**

Question 7 continued

TOTAL 10 MARKS

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8 (a) Using induction, prove that, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$$

(4)

(b) Use part (a) and standard results for series to show that

$$\sum_{r=1}^{n}(-r^2+4r-1)=\frac{1}{6}n(an^2+bn+c)$$

where a, b and c are constants to be found. **(3)**

(c) (i) Find the value of n for which $\sum_{r=1}^{n}(-r^2+4r-1)=0$. **(2)**

(ii) Hence, write down the values of n for which $\sum_{r=1}^{n}(-r^2+4r-1)>0$. **(1)**

Question 8 continued

Do not write outside the box

Question 8 continued

Do not write outside the box

Question 8 continued

TOTAL 10 MARKS

9 A particle is attached to the end of a spring and the system is immersed in a fluid. The system is connected to a forced oscillator which causes the system to oscillate.

The motion of the system can be modelled by the differential equation

$$2\frac{d^2x}{dt^2}+40\frac{dx}{dt}+128x=2\,e^{-2t}$$

where x m is the vertical displacement of the particle from its equilibrium position and t is the time, in seconds, after the motion begins.

The particle starts at equilibrium and is given an initial velocity of 3 m s^{-1}.

Find an expression for x in terms of t. **(9)**

Question 9 continued

TOTAL 9 MARKS

10 (a) Prove that the first four terms in the Maclaurin series of $\cos(x)$ are

$$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$$

(4)

(b) Hence, find an expression for the nth term in the Maclaurin series of $\cos(x)$. **(1)**

(c) Deduce that

$$x^5\cos(x^2)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+5}}{(2n)!}$$ **(1)**

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Question 10 continued

END OF PAPER **TOTAL 6 MARKS**

TOTAL FOR PAPER IS 75 MARKS