A Level
Further Maths

Bronze Set B, Core Pure 2 (Edexcel version)

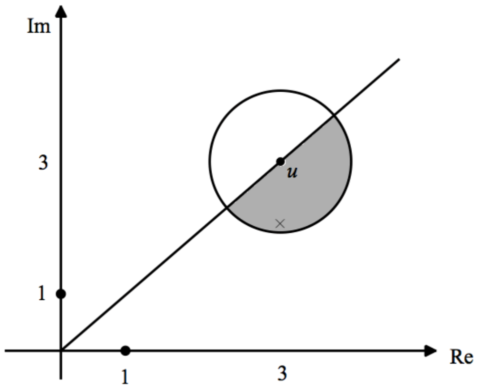
Question	Solution	Partial Marks	Guidance
1	Three equations for this problem are $13a+9b+10c=17080$ $8a+b+6c=8920$ $11a+10b+15c=21100$ In matrix form this is $\begin{pmatrix}13 & 9 & 10\\ 8 & 1 & 6\\ 11 & 10 & 15\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}17080\\ 8920\\ 21100\end{pmatrix}$ $\Rightarrow \begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}13 & 9 & 10\\ 8 & 1 & 6\\ 11 & 10 & 15\end{pmatrix}^{-1}\begin{pmatrix}17080\\ 8920\\ 21100\end{pmatrix}$ $\Rightarrow \begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}400\\ 320\\ 900\end{pmatrix}$ so price of laptop **A** is £400, price of laptop **B** is £320, price of laptop **C** is £900	B1 B1 M1(dep*) A1 **[4]**	Writes down at least one equation involving the price of each laptop (can be implied by their matrix form) Allow use of any variable Correct matrix equation of the system NB: a correct matrix equation of the system scores B1 B1 regardless of any other working Complete method to solve their matrix equation Solves the system to find the price of each laptop Must state price of laptop **A** = … , price of laptop **B** = … and price of laptop **C** = … or have defined their variables at some stage
2	$w=x+4 \Rightarrow x=w-4$ $5(w-4)^3-(w-4)^2+2(w-4)+1=0$ $\Rightarrow 5(w^3-12w^2+16w-64)-(w^2-8w+16)+2w-8+1=0$ $\Rightarrow 5w^3-61w^2+90w-343=0$	M1* M1(dep*) A1 A1 **[4]**	Substitutes their $w \pm 4$ into the equation Expands brackets and collects like terms Any two of the four coefficients correct Completely correct equation **NB:** if they substitute $x-4$ into their equation, this should be condoned and can still score 5/5

3	$x = r\cos\theta = 3\cos\theta + 2\sin\theta\cos\theta$ $\frac{dx}{d\theta} = -3\sin\theta + 2\cos 2\theta$ $-3\sin\theta + 2\cos 2\theta = 0 \Rightarrow -3\sin\theta + 2(1-2\sin^2\theta) = 0$ $\Rightarrow 4\sin^2\theta + 3\sin\theta - 2 = 0$ $\Rightarrow \sin\theta = \frac{-3+\sqrt{41}}{2(4)}$ so $OP = 3 + \frac{-3+\sqrt{41}}{4} = \frac{9+\sqrt{41}}{4}$	B1 M1[1] A1 M1[2](dep1) A1 M1(dep2) A1 **[7]**	Correct x in terms of θ Uses of product rule or chain rule on their x in terms of θ or y in terms of θ Obtains the correct equation Method to solve their quadratic for $\sin\theta$ Correct value of $\sin\theta$ (negative θ must be discounted or A0) Substitutes their $\sin\theta$ into the equation of the curve to find r Correct exact length
4	l_2 has equation $\mathbf{r} = (4\mathbf{i} + \mathbf{j}) + s(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ Note that l_1 and l_2 are not parallel $4 + 2s = 1 + 4t$ $1 - 3s = -2 - t$ $5s = 1 - t$ [Solves two of these and shows the three equations are not consistent – see notes] Hence, they are skew	B1 B1 M1 A1 A1 **[5]**	States or implies correct parametric form of l_2 States that the lines are not parallel at any stage Equates at least two components and solves for s or t One correct value of s or t (see below) $\begin{cases} s = \frac{3}{2} \\ t = \frac{3}{2} \\ \frac{15}{2} \neq -\frac{1}{2} \end{cases}$ $\begin{cases} s = \frac{1}{22} \\ t = \frac{17}{22} \\ \frac{19}{22} \neq -\frac{61}{22} \end{cases}$ $\begin{cases} s = \frac{1}{2} \\ t = -\frac{3}{2} \\ 5 \neq -5 \end{cases}$ Verify that all three lines fail to intersect and concludes that they are skew

5 (a)	$y = \cosh x = \dfrac{e^x + e^{-x}}{2}$ $\Rightarrow 2y = e^x + e^{-x}$ $\Rightarrow e^{2x} - 2ye^x + 1 = 0$ $\Rightarrow e^x = \dfrac{2y + \sqrt{4y^2 - 4}}{2}$ $\Rightarrow e^x = y + \sqrt{y^2 - 1}$ $\Rightarrow x = \ln(y + \sqrt{y^2 - 1})$ Hence $\operatorname{arcosh} x = \ln(x + \sqrt{x^2 - 1})$ **AG**	B1 M1* M1(dep*) A1 **[4]**	States or implies correct exponential definition of cosh(x) Re-arranges the equation into a 3TQ in e^x Complete method to solve for y in terms of x, including solution of their 3TQ and taking logs (allow them to keep the ± for this mark) Complete and convincing proof with no errors seen They can stop at $x = \ln(y + \sqrt{(y^2 - 1)})$ if they give a statement such as 'as required'
5 (b)	$\dfrac{d}{dx}(\operatorname{arcosh} x) = \dfrac{1}{x + \sqrt{x^2 - 1}} \times \left(1 + \dfrac{2x}{2\sqrt{x^2 - 1}}\right)$ $\Rightarrow \dfrac{d}{dx}(\operatorname{arcosh} x) = \dfrac{1}{x + \sqrt{x^2 - 1}} \times \dfrac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}$ $= \dfrac{1}{\sqrt{x^2 - 1}}$ as required	M1 A1 A1 **[3]**	Method to find the derivative with clear attempt at chain rule Correct unsimplified derivative Obtains the given answer convincingly with no errors seen

5 (c)	$\frac{dy}{dx}=\frac{12}{\sqrt{x^2-1}}-5$ At P, we have $\frac{12}{\sqrt{x^2-1}}-5=0$ $\Rightarrow \frac{12}{5}=\sqrt{x^2-1}$ $\Rightarrow x^2=\frac{144}{25}+1$ $\Rightarrow x=\frac{13}{5}$ so a = 13/5 then $y=12\operatorname{arcosh}\left(\frac{13}{5}\right)-5\left(\frac{13}{5}\right)$ $=12\ln\left(\frac{13}{5}+\sqrt{\left(\frac{13}{5}\right)^2-1}\right)-5\left(\frac{13}{5}\right)$ $=12\ln 5-13$ so b = 12, c = –13	M1 A1 M1* M1(dep*) A1 **[5]**	States correct derivative, sets it equal to zero and uses a complete method to solve for x (can be implied) Correct x coordinate of the maximum point or value of a stated Substitutes their x coordinate into the equation of the curve Uses the logarithmic definition of arcosh to find the exact value of y at their x coordinate Correct values of a, b and c stated with no errors seen
6 (a)	$\begin{pmatrix}7 & 24\\ 24 & -7\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$ $\Rightarrow 7x+24y=x$ or $24x-7y=y$ $\Rightarrow y=-\frac{1}{4}x$ or $y=3x$ and only (0, 0) satisfies both of these, (so (0,0) is the only invariant point under T)	M1 A1 **[2]**	Complete method to find the points invariant under T Shows that the only point that satisfies the condition for invariance is the origin NB: no marks for showing (0, 0) is an invariant point

6 (b)	$\begin{pmatrix} 7 & 24 \\ 24 & -7 \end{pmatrix}\begin{pmatrix} x \\ mx+c \end{pmatrix} = \begin{pmatrix} x' \\ mx'+c \end{pmatrix}$ $\Rightarrow \begin{pmatrix} 7x+24(mx+c) \\ 24x-7(mx+c) \end{pmatrix} = \begin{pmatrix} x' \\ mx'+c \end{pmatrix}$ $\Rightarrow 7x+24mx+24c = x'$ or $24x-7mx-7c = mx'+c$ $\Rightarrow 24x-7mx-7c = m(7x+24mx+24c)+c$ $\Rightarrow x(24m^2+14m-24)+(24mc+8c)=0$ This can only be true if both $24m^2+14m-24=0$ and $24mc+8c=0$ First equation $m=\frac{3}{4}$ or $-\frac{4}{3}$ Second equation $\Rightarrow m=-\frac{1}{3}$ or $c=0$ Combining these shows that $m \neq -\frac{1}{3}$, so only two cases left: $m=\frac{3}{4}, c=0 \Rightarrow y=\frac{3}{4}x$ and $m=-\frac{4}{3}, c=0 \Rightarrow y=-\frac{4}{3}x$	M1* M1**(dep*) A1 M1(dep**) A1 A1 **[6]**	Sets up a matrix equation to find invariant lines and extracts a pair of simultaneous equations Substitutes for x' and gathers terms in *x* Correct equation with terms in *x* gathered Deduces correct conditions for invariant lines using their equation with terms in *x* gathered Obtains correct invariant lines from correct workings (one mark for each correct line)
7 (a)	$(3+3\text{i})^2 + a(3+3\text{i}) + b = 0$ $\Rightarrow 9+18\text{i}-9+3a+3a\text{i}+b=0$ $\Rightarrow (3a+b)+(18+3a)\text{i}=0$ Comparing real and imaginary parts gives ***a* = –6** from second equation and thus ***b* = 18** from second equation	M1* M1(dep*) A1 **[3]**	Substitutes *u* into the quadratic and expands the brackets using $\text{i}^2 = -1$ Collects and compares real and imaginary parts, obtaining two equations in *a* and *b* which they should attempt to solve Correct value of *a* and correct value of *b*

7 (a) ALT	3 – 3i is also a solution hence $x = 3 \pm 3\text{i} \Rightarrow (x-3)^2 = -9$ $\Rightarrow x^2 - 6x + 18 = 0$ so $a = -6$ and $b = 18$	M1* M1(dep*) A1 **[3]**	States or implies that 3 – 3i is also a solution Complete method to use information about the roots to find the quadratic with these roots (see below for more alts) Correct value of a and correct value of b **ALTs (more alternatives for the 2nd M1):** • Writes down $(x-(3+3\text{i}))(x-(3-3\text{i}))$ and expands the brackets using $\text{i}^2 = -1$ (all this required for 2nd M1) • Works out sum and product of roots and using $\text{i}^2 = -1$ when computing the product (all this required for 2nd M1)
7 (b)		B1 **[1]**	All three complex numbers shown on an Argand diagram as points or vectors in relatively correct positions Detailed scale not required and labels not required See diagram in part (c) (allow i for 1 labelled on Im axis)
7 (c)		B1 B1 B1 **[3]**	For a circle of radius 1 centered at u (must **not** cross the axes) Perpendicular bisector of 1 and i seen (this is the line Re = Im) Correct region shaded **Note:** do not penalise if they draw on a separate Argand diagram to (b), as this requirement was primarily to help them with (d)

7 (d)	Arg(z) is least at the point where the line from O to the circle is tangent Using Pythagoras', we have $\lvert z\rvert = \sqrt{\lvert u\rvert^2 - 1^2} = \sqrt{17}$	M1* M1(dep*) A1 **[3]**	Identification of the critical point where the argument is least (can be implied) Complete method to find the modulus of z at this critical point Correct modulus
8 (a)	When $n = 1$, LHS = 1 RHS = 1/2 (1 + 1) = 1 Since LHS = RHS, the statement is true for $n = 1$ Assume true for $n = k$, i.e. $\sum_{r=1}^{k} r = \frac{k}{2}(k+1)$ Then for $n = k + 1$, we have $\sum_{r=1}^{k+1} r = \sum_{r=1}^{k} r + (k+1) = \frac{k}{2}(k+1) + (k+1)$ $= (k+1)\left(\frac{k}{2}+1\right)$ $= \frac{(k+1)}{2}(k+2)$ $= \frac{(k+1)}{2}((k+1)+1)$ and so the statement is true for $n = k + 1$ If <u>true for $n = k$</u>, then it has been shown by induction to be <u>true for $n = k + 1$</u>. Since <u>true for $n = 1$</u>, it <u>is true for all positive integers n</u>	B1 M1 A1 A1 **[4]**	Convincingly shows the statement is true for $n = 1$ Makes the assumption and uses it to find an expression for the sum when $n = k + 1$ (which should be correctly partitioned) Convincingly shows the statement is true for $n = k + 1$ If they stop at the penultimate line, they must make it clear that this is the form required: 'hence true for $n = k + 1$', 'as required', etc. is OK Complete and convincing proof with no errors seen and conclusion with all the underlined elements (or equivalent)

8 (b)	$\sum_{r=1}^{n}(-r^2+4r-1)=-\sum_{r=1}^{n}r^2+4\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$ $=-\frac{n}{6}(n+1)(2n+1)+2n(n+1)-n$ $=\frac{n}{6}(-2n^2-3n-1+12n+12-6)$ $=\frac{n}{6}(-2n^2+9n+5)$ so $a = -2$, $b = 9$ and $c = 5$	M1* M1(dep*) A1 cao **[3]**	Correctly partitions the sum Substitutes in the correct formula for each partitioned element and attempts to take out a factor of $n/6$ (allow up to one coefficient error) Obtains the correct answer in the required form or values of a, b and c stated with sufficient working
8 (c) (i)	$-2n^2+9n+5=0 \Rightarrow (2n+1)(n-5)=0$ so $n = 5$	M1 A1 **[2]**	Attempts to find the value of n for which the sum is 0 Correct n Correct answer scores 2/2 regardless of method
8 (c) (ii)	$1 \le n < 5$	B1 **[1]**	Correct range. Allow $n = 1, 2, 3, 4$

9	Auxiliary equation is $2m^2 + 40m + 128 = 0$ $\Rightarrow m = -4, m = -16$ so solution to homogenous equation is $x = A\mathrm{e}^{-4t} + B\mathrm{e}^{-16t}$ Particular integral of the form $x = C\mathrm{e}^{-2t}$. Putting this in gives: $2(-2)^2 C\mathrm{e}^{-2t} + 40(-2)C\mathrm{e}^{-2t} + 128C\mathrm{e}^{-2t} = 2\mathrm{e}^{-2t}$ $\Rightarrow 8C - 80C + 128C = 2$ $\Rightarrow C = \frac{1}{28}$ so general solution to the differential equation is $x = A\mathrm{e}^{-4t} + B\mathrm{e}^{-16t} + \frac{1}{28}\mathrm{e}^{-2t}$ The particle starts at equilibrium so $A + B = -\frac{1}{28}$ Velocity of the particle is $\frac{dx}{dt} = -4A\mathrm{e}^{-4t} - 16B\mathrm{e}^{-16t} - \frac{1}{14}\mathrm{e}^{-2t}$ Initial velocity is 3 so $-4A - 16B = \frac{43}{14}$ Solving the two equations gives $A = \frac{5}{24}, B = -\frac{41}{168}$ $\Rightarrow x = \frac{5}{24}\mathrm{e}^{-4t} - \frac{41}{168}\mathrm{e}^{-16t} + \frac{1}{28}\mathrm{e}^{-2t}$	M1* M1 A1 M1* A1 A1ft M1(dep*) A1 A1 **[9]**	States correct auxiliary equation to the curve States solution to homogenous part of the form $x = A\mathrm{e}^{pt} + B\mathrm{e}^{qt}$, $p \neq q$ Correct solution to homogenous part of the equation Identifies the correct form of the particular solution and substitutes it into the curve with complete method to find 'C'. Allow one error in differentiation Correct particular integral Brings their particular integral and solution to the homogenous equation together to write the general solution for x in terms of t Uses the initial conditions to write down one correct equation in terms of 'A' and 'B' ft their solution Both equations for A and B correctly obtained Solves the equations for A and B correctly and writes down the correct solution to the differential equation with x in terms of t. No errors seen

10 (a)	$\cos 0 = 1$ $\frac{d}{dx}(\cos x) = -\sin x$ which evaluates to 0 at 0 $\frac{d^2}{dx^2}(\cos x) = -\cos x$ which evaluates to -1 at 0 $\frac{d^3}{dx^3}(\cos x) = \sin x$ which evaluates to 0 at 0 $\frac{d^4}{dx^4}(\cos x) = \cos x$ which evaluates to 1 at 0 $\frac{d^5}{dx^5}(\cos x) = -\sin x$ which evaluates to 0 at 0 $\frac{d^6}{dx^6}(\cos x) = -\cos x$ which evaluates to -1 at 0 Hence $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$	B1 M1 A1 A1 **[4]**	States that cos(0) = 1. This can be implied if they make it clear (perhaps from quoting the general series) that the first term arises from evaluating the function at 0 Attempts to compute at least two derivatives of cos$x$ and evaluate them at 0 Four derivatives of cos(x) correctly evaluated at 0 Correct series expansion for cos(x) obtained convincingly Full marks for a solution that comments e.g. 'all odd derivatives of cos$x$ are ±sin$x$ which are 0 at 0' and doesn't actually compute them
10 (b)	nth term is $\frac{(-1)^n x^{2n}}{(2n)!}$	B1 **[1]**	Correct nth term
10 (c)	$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ so $\cos(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$ $\Rightarrow x^5 \cos(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+5}}{(2n)!}$	B1 **[1]**	Convincing proof We need to see one intermediate step and this will usually be the series for cos(x^2)