Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				

Examiner Comments	Total Marks

FURTHER MATHEMATICS

A LEVEL CORE PURE 1

CM

Bronze Set B (Edexcel Version)

Time allowed: 1 hour and 30 minutes

Instructions to candidates:

• In the boxes above, write your centre number, candidate number, your surname, other names and signature.

• Answer ALL of the questions.

• You must write your answer for each question in the spaces provided.

• You may use a calculator.

Information to candidates:

• Full marks may only be obtained for answers to ALL of the questions.

• The marks for individual questions and parts of the questions are shown in round brackets.

• There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to candidates:

• You should ensure your answers to parts of the question are clearly labelled.

• You should show sufficient working to make your workings clear to the Examiner.

• Answers without working may not gain full credit.

A2/FM/CP1

1 1 3 3 1 3 1 2 8 0 0 0 4

1 The 2×2 matrix $\mathbf{M}$ represents a reflection in the line $y = x$.

(a) Write down the matrix $\mathbf{M}$. **(1)**

The matrix $\mathbf{N}$ is defined such that

$$\mathbf{N} = \begin{pmatrix} 4 & 5 \\ 3 & k \end{pmatrix}$$

where k is a constant.

The singular matrix $\mathbf{R} = \mathbf{MN}$.

(b) Find the value of k. **(4)**

Question 1 continued

TOTAL 5 MARKS

Do not write outside the box

2
$$f(z) = z^3 + pz^2 + 11z + q$$

where p and q are real constants.

Given that the equation $f(z) = 0$ has positive roots

$$3,\ \frac{1}{\alpha} \text{ and } \left(4 - \frac{2}{\alpha}\right)$$

(a) (i) show that $\alpha^2 + \alpha - 2 = 0$,

(ii) and hence solve $f(z) = 0$ completely. **(5)**

(b) Find the values of p and q. **(3)**

Question 2 continued

TOTAL 8 MARKS

3 The curve C has the equation $y = \sin^2 x\sqrt{\cos x}$.

The region R is bounded by the curve C, the x-axis and the lines $x = 0$ and $x = 1$. The region R is rotated 2π radians about the x-axis to form the solid S.

Find, to three significant figures, the volume of the solid S. **(5)**

Do not write outside the box

Question 3 continued

TOTAL 5 MARKS

4 The complex number z is defined such that

$$z = \frac{-2+4\mathrm{i}}{3-\mathrm{i}}$$

(a) Showing all of your working clearly, find the modulus and argument of z.
Give the modulus as an exact value. **(5)**

Using de Moivre's theorem,

(b) find z^3, **(2)**

(c) find the values of w such that $w^3 = z$ **(3)**

Question 4 continued

Question 4 continued

Question 4 continued

TOTAL 10 MARKS

5 A company is making an investment proposition which they will pitch to an investor. They will require money from the investor each year for a certain number of years.

The company models the amount of investment they will require from the investor each year by the formula

$$P_r = \frac{12}{(r+3)(r+4)}$$

where P_r is the amount of money required, in hundreds of thousands of pounds, from the the investor in year r. In the model, the investor makes his initial investment in Year 1.

(a) Show that, according to the model, the total amount of money, in hundreds of thousands, given to the company by the investor in the first n years is given by

$$M_n = \frac{kn}{n+4}$$

where k is a constant to be determined. **(6)**

The company expect the investor's final payment in Year 5.

The company predicts they will first enter into profit in Year 5. They expect to make £110 000 profit in Year 5. It is expected that the profit made by the company each year will then increase arithmetically by £30 000.

The investor will receive 35% of the yearly profit made by the company.

(b) Determine, according to the model, the year in which the investor will get his entire investment back. Show your method. **(4)**

Question 5 continued

Question 5 continued

Question 5 continued

TOTAL 10 MARKS

6 (a) **Prove** that

$$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

where C is an arbitrary constant. **(3)**

(b) Using the substitution $u = \sinh x$, show that

$$\int_0^a \frac{\cosh x}{1+2\sinh^2 x}\,dx = \frac{1}{k}\arctan(k\sinh a)$$

where k is a constant to be found. **(4)**

(c) (i) Write down the limiting value of $\sinh a$ as $a \to \infty$. **(1)**

(ii) Hence, evaluate

$$\int_0^\infty \frac{\cosh x}{1+2\sinh^2 x}\,dx$$

(1)

Question 6 continued

Do not write outside the box

Question 6 continued

Do not write outside the box

Question 6 continued

TOTAL 9 MARKS

7 The straight line l passes through the points A and B whose position vectors are $2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{k}$ respectively. The plane Π_1 has the equation $\mathbf{r} \cdot (2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}) = -14$.

(a) Given that l intersects Π_1, find the position vector of the intersection point. **(5)**

The plane Π_2 contains the line l and is perpendicular to Π_1.

(b) Show that the vector $7\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ is normal to Π_2. **(3)**

(c) Hence, find the equation of the plane Π_2, giving your answer in the form $ax + by + cz = 1$, where a, b and c are constants to be determined. **(3)**

Question 7 continued

TOTAL 11 MARKS

8 Prove by induction that, for $n \in \mathbb{N}$,

$$f(n) = 3^{2n} - 1$$

is divisible by 8. **(6)**

Do not write outside the box

Question 8 continued

TOTAL 6 MARKS

9 The RC circuit is an electrical circuit which contains a resistor and a capacitor. Figure 1 shows an RC circuit of interest. The resistor has resistance R and the capacitor has capacitance C, where $R > 0$ and $C > 0$.

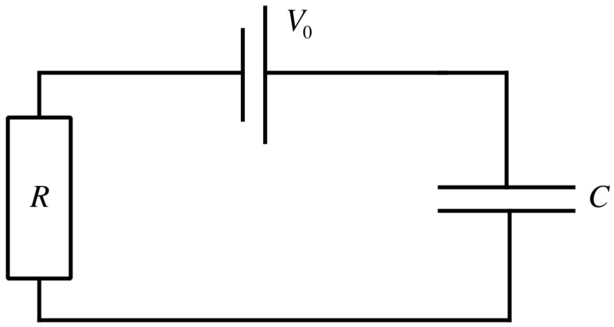

Figure 1

The voltage across the battery is a constant V_0.

The charge on the capacitor Q can be shown to satisfy the differential equation

$$R\frac{dQ}{dt} + \frac{1}{C}Q = V_0$$

The initial charge on the capacitor is 0.

(a) By solving the differential equation directly,

$$Q = V_0 C\left(1 - e^{-kt}\right)$$

where k is a constant in terms of R and C and should be determined. **(6)**

The current I in the circuit is given by $I = \frac{dQ}{dt}$.

(b) Using your answer to part (a), find an expression for I in terms of t. **(2)**

(c) Sketch, on separate axes,

(i) the graph of Q against t

(ii) the graph of I against t

On your sketch, show the coordinates of any points where the graphs cross or meet the coordinate axes and the equations of any asymptotes. **(3)**

Do not write outside the box

Question 9 continued

Question 9 continued

Do not write outside the box

Question 9 continued

END OF PAPER **TOTAL 11 MARKS**

TOTAL FOR PAPER IS 75 MARKS