A Level
Further Maths

Bronze Set B, Core Pure 1 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1 (a)	$\mathbf{M}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	B1 **[1]**	Correct matrix **M**
1 (b)	$\mathbf{R}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}4 & 5\\ 3 & k\end{pmatrix}=\begin{pmatrix}3 & k\\ 4 & 5\end{pmatrix}$ $\det\mathbf{R}=15-4k$ Since **R** is singular, det**R** = 0, so 15 – 4k = 0 ⇒ k = … $k=\frac{15}{4}$	B1ft M1* M1(dep*) A1 **[4]**	Correct matrix **R** ft their (a) Attempts to work determinant of their matrix **R** Sets their det**R** = 0 and solves for k Correct value of k
2 (a) (i)	$11=3\left(\frac{1}{\alpha}\right)+3\left(4-\frac{2}{\alpha}\right)+\frac{1}{\alpha}\left(4-\frac{2}{\alpha}\right)$ $\Rightarrow 11=\frac{3}{\alpha}+12-\frac{6}{\alpha}+\frac{4}{\alpha}-\frac{2}{\alpha^2}$ Multiplying through by α^2 gives $11\alpha^2=3\alpha+12\alpha^2-6\alpha+4\alpha-2$ $\Rightarrow \alpha^2+\alpha-2=0$ **AG**	M1* M1(dep*) A1 **[3]**	Forms correct equation Condone –11 on LHS Method to re-arrange the equation into a 3TQ Obtains the given answer convincingly and with no errors seen
2 (a) (ii)	$\Rightarrow(\alpha+2)(\alpha-1)=0$ Require roots to be positive, so $\alpha>0\Rightarrow\alpha=1$ And the roots are 3, 1 and 2	M1 A1 **[2]**	Complete method to solve the 3TQ and use at least one of their values of α to find the roots of the equation (positive or not) Correct roots of the equation
2 (b)	Product of roots is 6, so q = –6 Sum of roots $3+2+1=6$, so p = –6	M1 A1 A1 **[3]**	Method to find either p or q (allow sign confusion) Correct value of p or q Correct value of p and correct value of q

3	Volume of $S = \pi\int_0^1 \sin^4 x \cos x\, dx$ $= \pi\left[\frac{\sin^5 x}{5}\right]_0^1$ $= \pi\left[\frac{\sin^5 1}{5} - \frac{\sin^5 0}{5}\right]$ $= 0.265$	M1* M1**(dep*) A1 M1(dep**) A1 **[5]**	Correct expression for the volume seen (unsimplified or better) Allow omission of π for this mark States integral of the form $k\sin^5 x$ Obtains correct integral Allow omission of π for this mark Substitutes limits into their integral in the correct order Obtains the correct answer cao
4 (a)	$z = \frac{-2+4\mathrm{i}}{3-\mathrm{i}} \times \frac{3+\mathrm{i}}{3+\mathrm{i}}$ $= \frac{-6-2\mathrm{i}+12\mathrm{i}-4}{10}$ $= -1+\mathrm{i}$ $\lvert z\rvert = \sqrt{(-1)^2+1^2} = \sqrt{2}$ $\mathrm{Arg}z = \pi - \tan^{-1}\left(\frac{1}{1}\right) = \frac{3\pi}{4}$	M1 A1 A1ft M1 A1ft **[5]**	Complete method to realise denominator by multiplying numerator and denominator by 3 + i. We need to see them attempt the expansion of the numerator and denominator and use of $\mathrm{i}^2 = 1$ Obtains $z = -1 + \mathrm{i}$ Correct modulus ft their z as an exact value Complete method to find the argument of their z Correct argument ft their z
4 (a) **ALT**	$\lvert z\rvert = \frac{\lvert -2+4\mathrm{i}\rvert}{\lvert 3-\mathrm{i}\rvert} = \frac{\sqrt{2^2+4^2}}{\sqrt{3^2+1^2}} = \sqrt{2}$ $\arg(z) = \arg(-2+4\mathrm{i}) - \arg(3-\mathrm{i})$ $= \left(\pi - \tan^{-1}\left(\frac{4}{2}\right)\right) - \left(-\tan^{-1}\left(\frac{1}{3}\right)\right)$ $= \frac{3}{4}\pi$	M1 A1 M1 (scheme change) M1 A1 **[5]**	Attempts to find modulus of z with correct attempt at working out the modulus of either numerator or denominator seen Correct modulus of z States that $\arg(z) = \arg(-2 + 4\mathrm{i}) - \arg(3 - \mathrm{i})$ Complete method to find the argument of the numerator or denominator Correct argument of z

4 (b)	$z=\sqrt{2}\left(\cos\left(\frac{3}{4}\pi\right)+\text{i}\sin\left(\frac{3}{4}\pi\right)\right)$ Hence $z^3=2\sqrt{2}\left(\cos\left(\frac{9}{4}\pi\right)+\text{i}\sin\left(\frac{9}{4}\pi\right)\right)$	M1 A1 **[2]**	Attempts to use their modulus and argument of z and de Moivre's theorem to find the modulus and argument of z^3 Obtains z^3 correctly in any form, e.g. 2 + 2i Correct answer gets 2/2 (even if they don't use de Moivre's theorem), but no method marks for any other method, e.g. binomial expansion
4 (c)	$r^3=\sqrt{2}\Rightarrow r=(\sqrt{2})^{\frac{1}{3}}$ $3\theta=\frac{3}{4}\pi,\ \frac{11}{4}\pi$ or $\frac{19}{4}\pi$ $\Rightarrow\theta=\frac{1}{4}\pi,\ \frac{11}{12}\pi,\ \frac{19}{12}\pi$ so $w=(\sqrt{2})^{\frac{1}{3}}\left(\cos\left(\frac{1}{4}\pi\right)+\text{i}\sin\left(\frac{1}{4}\pi\right)\right)$ or $w=(\sqrt{2})^{\frac{1}{3}}\left(\cos\left(\frac{11}{12}\pi\right)+\text{i}\sin\left(\frac{11}{12}\pi\right)\right)$ or $w=(\sqrt{2})^{\frac{1}{3}}\left(\cos\left(\frac{19}{12}\pi\right)+\text{i}\sin\left(\frac{19}{12}\pi\right)\right)$	M1 A1 A1 **[3]**	Clear attempt at both r and θ with at least one θ correct Two values of θ correct [Any range is permissible. You may see $\theta=-\frac{5}{12}\pi,\ \frac{3}{4}\pi,\ \frac{11}{4}\pi$] Correct roots

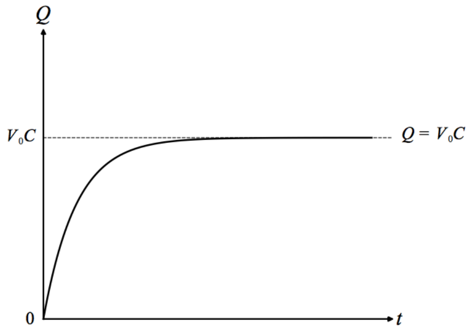
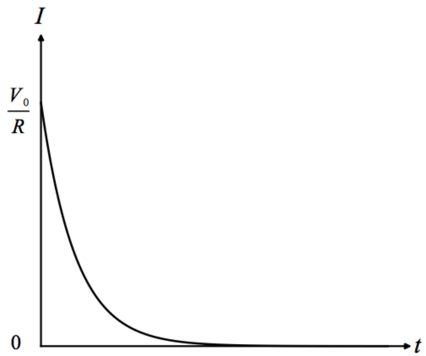
5 (a)	$P_r = \frac{A}{(r+3)} + \frac{B}{(r+4)} \Rightarrow 12 = A(r+4) + B(r+3)$ Setting $r = -4$ gives $B = -12$ Setting $r = -3$ gives $A = 12$	M1*	Attempts to express P_r in partial fractions (can be implied from correct partial fractions)
	So $P_r = \frac{12}{r+3} - \frac{12}{r+4}$	A1	Correct P_r in partial fractions (can be implied)
	Then $M_r = \sum_{r=1}^{n} P_r$ $= \left(\frac{12}{4} - \frac{12}{5}\right) + \left(\frac{12}{5} - \frac{12}{6}\right) + \ldots + \left(\frac{12}{n+2} - \frac{12}{n+3}\right) + \left(\frac{12}{n+3} - \frac{12}{n+4}\right)$	M1(dep*) A1	Writes out the series starting from $r = 1$ or 0 with at least three terms shown, including first term and the last term, using their P_r Correct unsimplified series
	$= 3 - \frac{12}{n+4}$ $= \frac{3n+12-12}{n+4}$	A1ft	Cancels all terms from their series ft their partial fractions Their series must actually telescope for this mark, i.e. they need a minus sign in their partial fractions, and start from $r = 1$
	$= \frac{3n}{n+4}$	A1 **[6]**	Obtains the correct simplified value of the series convincingly or k stated with sufficient working

5 (b)	Total money given by investor is $M_5 = \frac{3(5)}{5+4} = \frac{15}{9}$ (hundreds of thousands of pounds) In t years the investor will make $0.35 \times \frac{t}{2}(2.4 + (t-1)(0.3))$ from the company (hundreds of thousands of pounds) Require $\frac{15}{9} = 0.35 \times \frac{t}{2}(2.4 + (t-1)(0.3))$ $\Rightarrow 0.3t^2 + 2.1t - \frac{200}{21} = 0$ $\Rightarrow t = 3.132...$ so investor will make his money back in Year 8	B1ft M1* M1(dep*) A1 **[4]**	Works out M_5 using their (b). Allow 15/9 or (£)166,666.67 ft their (b) Sight of 0.4 × sum of first t years of the arithmetic series. Allow 120,000 instead of 1.2 etc (accept any letter) ***OR***: states that investor makes £38,500 in Y5 and £49,000 in Y6 Sets up an equation **with consistent units** and employs a complete method to find t ***OR***: complete attempt to use trial and error to find the year in which the investor makes his return Correct year Allow 'at the end of Year 8 / beginning of Year 9'
6 (a)	Let $x = \tan u$, then $dx = \sec^2 u\, du$ So $\int \frac{1}{1+x^2}dx = \int \frac{1}{1+\tan^2 u}(\sec^2 u)du$ $= \int du$ $= u + C$ $= \arctan x + C$ **AG**	M1 M1 A1 **[3]**	Substitutes in $x = \tan u$ and uses $dx = \sec^2 u\, du$ Uses $1 + \tan^2 u = \sec^2 u$ Obtains the given result convincingly with no errors seen
6 (b)	$\int_0^{\sinh a} \frac{\cosh x}{1+2u^2}\left(\frac{1}{\cosh x}\right)du = \int_0^{\sinh a} \frac{1}{1+2u^2}du$ $= \frac{1}{\sqrt{2}}\left[\arctan(u\sqrt{2})\right]_0^{\sinh a}$ $= \frac{1}{\sqrt{2}}\arctan(\sqrt{2}\sinh a)$	M1* B1 M1(dep*) A1 **[4]**	Substitutes in $u = \sinh u$ and replaces dx correctly Correct limits States integral of the form $(\arctan(pu))/p$ where p is any constant Obtains the correct result convincingly
6 (c) (i)	∞	B1 **[1]**	Correct limit

6 (c) (ii)	$= \lim_{a\to\infty}\left(\frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\sinh a\right)\right)$ $= \frac{1}{\sqrt{2}}\lim_{p\to\infty}\arctan(p)$ $= \frac{1}{\sqrt{2}}\left(\frac{\pi}{2}\right)$ $= \frac{\pi}{2\sqrt{2}}$	B1 **[1]**	Correct limit (no working necessary)
7 (a)	$\overrightarrow{AB} = \begin{pmatrix}1\\0\\1\end{pmatrix} - \begin{pmatrix}2\\1\\-1\end{pmatrix} = \begin{pmatrix}-1\\-1\\2\end{pmatrix}$ so equation of l is e.g. $\mathbf{r} = \begin{pmatrix}2\\1\\-1\end{pmatrix} + \lambda\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ Want $\begin{pmatrix}2-\lambda\\1-\lambda\\-1+2\lambda\end{pmatrix}\cdot\begin{pmatrix}2\\5\\-3\end{pmatrix} = -14$ $\Rightarrow 2(2-\lambda)+5(1-\lambda)-3(-1+2\lambda) = -14$ $\Rightarrow -13\lambda = -26$ $\Rightarrow \lambda = 2$ so point is $(0, -1, 3)$	M1* A1 M1(dep*) A1ft A1 **[5]**	Complete method to find the equation of the line l A correct equation for the line l Employs a complete method to find the position vector of the intersection, leading to λ = … Correct value of their parameter ft their l Correct point

7 (b)	$\begin{pmatrix}7\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}-1\\-1\\2\end{pmatrix}=-7+1+6=0$, so 7**i** – **j** + 3**k** is perpendicular to the line l $\begin{pmatrix}7\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}2\\5\\-3\end{pmatrix}=14-5-9=0$, so 7**i** – **j** + 3**k** is parallel to Π_1 Hence 7**i** – **j** + 3**k** is perpendicular to Π_2	M1 A1 A1 **[3]**	Attempts to show that 7**i** – **j** + 3**k** is perpendicular to l (ft their direction vector) ***OR*** the plane Π_1 Shows that 7**i** – **j** + 3**k** is perpendicular to l (ft their direction vector) ***OR*** parallel to the plane Π_1 Shows that 7**i** – **j** + 3**k** is both perpendicular to l (ft their direction vector) and the normal to the plane Π_1 + conclusion, e.g. 'hence perp. to Π_2', 'as required', 'qed', etc.
7 (b) ALT	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-1 & -1 & 2\\2 & 5 & -3\end{vmatrix}=(3-10)\mathbf{i}-(3-4)\mathbf{j}+(-5+2)\mathbf{k}$ $=-7\mathbf{i}+\mathbf{j}-3\mathbf{k}$ And 7**i** – **j** + 3**k** is a multiple of this, so it is perpendicular to Π_2	M1 A1 A1 **[3]**	Method to find (–**i** – **j** + 2**k**) × (2**i** + 5**j** – 3**k**) (or other way round) ***OR*:** states one of $-a-b+2c=0$ or $2a+5b-3c=0$ At least two correct coefficients from their cross product ***OR*:** solves for at least one ratio Complete and convincing proof with no errors seen (conclusion is only necessary if they don't obtain 7**i** – **j** + 3**k** directly from the cross/dot product)
7 (c)	$7(2)-1(1)+3(-1)=10$ ***OR*** $7(1)-1(0)+3(1)=10$ so equation of plane is $7x-y+3z=10\Rightarrow\frac{7}{10}x-\frac{1}{10}y+\frac{3}{10}z=1$	M1 A1ft A1 **[3]**	Method to find 'd' Correct equation in any form (ft is allowed if they use their (a)) Obtains correct equation of the plane in the required form cao

8	$f(1) = 3^2 - 1 = 8$ which is divisible by 8, so true for $n = 1$ Assume true for $n = k$, i.e. $f(k) = 3^{2k} - 1$ is divisible by 8 Then $f(k+1) - f(k) = 3^{2k+2} - 1 - (3^{2k} - 1)$ $= 9(3^{2k}) - 1 - 3^{2k} + 1$ $= 8(3^{2k})$ so $f(k+1) = f(k) + 8(3^{2k})$. Hence $f(k + 1)$ is divisible by 8 and the statement is true for $n = k + 1$ If true for $n = k$, then it has been shown by induction to be true for $n = k + 1$. Since true for $n = 1$, it is true for all positive integers n	B1 M1* M1(dep*) M1(dep*) A1 A1 **[6]**	Convincingly shows the statement is true for $n = 1$ 'So true for $n = 1$' can be implied if given in final conclusion Makes the assumption and writes down $f(k + 1)$ Beings complete method to connect $f(k + 1)$ with $f(k)$ Uses $3^{2k + \text{'their 2'}} = p(3^{2k})$ Shows the statement is true for $n = k + 1$ Complete and convincing proof with no errors seen and a conclusion with all the underlined elements A0 if they write 'true for all integers n'
9 (a)	Dividing by RC gives $\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_0}{R}$ Integrating factor is $e^{\int \frac{1}{RC} dt} = e^{\frac{t}{RC}}$ $\Rightarrow Qe^{\frac{t}{RC}} = \frac{V_0}{R}\int e^{\frac{t}{RC}}\,dt$ $\Rightarrow Qe^{\frac{t}{RC}} = V_0 C\,e^{\frac{t}{RC}} + k$ Initial charge is 0, so $0 = V_0 C\,e^0 + k \Rightarrow k = -V_0 C$ Hence solution is $Qe^{\frac{t}{RC}} = V_0 C\,e^{\frac{t}{RC}} - V_0 C$ $\Rightarrow Q = V_0 C - V_0 C\,e^{-\frac{t}{RC}} = V_0 C(1 - e^{-\frac{t}{RC}})$	 B1 M1* A1 A1 M1(dep*) A1 cso **[6]**	 Correct integrating factor For $Q \times$ their integrating factor $= \int (V_0/R) \times$ their integrating factor Correct expression in terms of integral Integrates correctly to get unsimplified expression including a constant Uses initial conditions to find the constant Correct expression for Q in required form ISW No marks if they substitute in Q, equate coefficients, etc.

9 (b)	$I = \frac{1}{RC}(V_0C)e^{-\frac{t}{RC}} = \frac{V_0}{R}e^{-\frac{t}{RC}}$	M1 A1 **[2]**	Attempts to differentiate their expression for Q wrt t Correct expression for I in terms of t
9 (c) **(i)** **(ii)**	 (asymptote at $Q = V_0C$)  (asymptote at $I = 0$/t-axis)	B1 B1 B1 **[3]**	**Mark parts (i) and (ii) together** Shape of graph of Q correct Shape of graph of I correct Correct intersection points and equations of asymptotes clearly stated on graphs of I and Q Give the 3rd B1 if the t axis asymptote of I is not stated explicitly but it is **clear** from the graph that the t axis is an asymptote