AS Level
Further Maths

Bronze Set A, Paper F1 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1	$5 = \frac{y_1 - y_{-1}}{2(0.5)} \Rightarrow y_1 - y_{-1} = 5$	B1	Correct equation in any form seen or implied
	$3\sin(4) + 1^2(5) = \frac{y_1 - 2(1) + y_{-1}}{0.5^2}$ $\Rightarrow y_1 + y_{-1} = 2.6823...$	B1 M1	Correct value of second derivative seen or implied Uses their value of the second derivative to form a (second) equation
	Subtracting the equations, we find $-2y_{-1} = 2.3176...$ $\Rightarrow y_{-1} = -1.16$	M1 A1 **[5]**	Brings together the information to find y_{-1} from their equations Correct value of y to 2 dp. Cao
2	$\frac{x+2-x}{x+2} < \frac{4}{x+1}$ $\Rightarrow 2(x+1)^2(x+2) < 4(x+1)(x+2)^2$ $\Rightarrow 2(x+1)(x+2)\left[(x+1) - 2(x+2)\right] < 0$ $\Rightarrow -2(x+1)(x+2)(x+3) < 0$	M1* A1 M1(dep*)	Attempts to multiply by $(x+1)^2(x+2)$ Correct unsimplified expression Attempts to re-arrange to find the CVs
	So CVs are –1, –2, –3 Then solution set is $\{x \in \mathbb{R} : -3 < x < 2\} \cup \{x \in \mathbb{R} : x > -1\}$	A1 A1 A1 **[6]**	Correct CVs (1 mark for 1 correct, 2 marks for all three correct) Correct solution set oe cso
3 (a)	$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 2 & -1 \\ -1 & 3 & 2 \end{vmatrix}$ $= (4 - -3)\mathbf{i} - (8 - 1)\mathbf{j} + (12 - -2)\mathbf{k}$ $= 7\mathbf{i} - 7\mathbf{j} + 14\mathbf{k}$	M1 A1 A1 **[3]**	Complete method to find the cross-product Method to find **b** x **a** is SCM1 A0 A0 Two components correct All three components correct

3 (b)	$\left\|(7\mathbf{i}-7\mathbf{j}+14\mathbf{k})\bullet(k\mathbf{i}-\mathbf{j})\right\|=10$ $\left\|7k+7\right\|=10$ $\Rightarrow 7k+7=\pm 10$ $\Rightarrow k=\frac{3}{7}$ or $k=-\frac{17}{7}$	M1 A1 A1 A1 **[4]**	Uses triple scalar product to form an equation. Allow the M1 if the modulus sign is omitted Obtains $7k+7$ One solution correct Both solutions correct
4 (a)	At $T=0$, $R=14.807\ldots$, $F=19.106\ldots$ so curve 2 corresponds to the foxes and curve 1 corresponds to the rabbits	B1* B1(dep*) **[2]**	Attempts to find population of rabbits and foxes at $T=0$ Correct conclusion Accept just 'curve 2 corresponds to the foxes' or 'curve 1 corresponds to the rabbits'
4 (b)	$R=\frac{48+29\left(\frac{1-t^2}{1+t^2}\right)-3\left(\frac{2t}{1+t^2}\right)}{3+2.2\left(\frac{1-t^2}{1+t^2}\right)+0.5\left(\frac{2t}{1+t^2}\right)}$ $=\frac{48(1+t^2)+29(1-t^2)-3(2t)}{3(1+t^2)+2.2(1-t^2)+0.5(2t)}$ $=\frac{48+48t^2+29-29t^2-6t}{3+3t^2+2.2-2.2t^2+t}$ $=\frac{19t^2-6t+77}{0.8t^2+t+5.2}$ **AG**	M1* M1(dep*) A1 **[3]**	Replaces cos$T$ and sin$T$ by the **correct** t formulae For getting to this stage (or better) Convincing proof with no errors seen
4 (c)	$25=\frac{19t^2-6t+77}{0.8t^2+t+5.2}$ $\Rightarrow 20t^2+25t+130=19t^2-6t+77$ $\Rightarrow t^2+31t+53=0$ $\Rightarrow t=-1.816...,\ t=-29.183...$ Principal values of $\frac{T}{2}$ are hence –1.067... and –1.536… So third time where $R=25$ is $2(-1.536+2\pi)=9.494...$	M1 A1 M1* M1(dep*) A1 **[5]**	Sets 25 equal to (b). 25000 equal to (b) is M0 Forms correct 3TQ Method to solve their 3TQ which results from setting 25 **OR** 25000 equal to (b) Calculates at least one of their principal values and uses this to find at least one value of T in range. Can be implied by a correct answer Correct time. Awrt 9.5

4 (d)	$23 = 5 + \dfrac{13.7t^2 + 22t + 66.3}{1.3t^2 + 3t + 4.2}$ $\Rightarrow 23.4t^2 + 54t + 75.6 = 13.7t^2 + 22t + 66.3$ $\Rightarrow 9.7t^2 + 32t + 9.3 = 0$ But $32^2 - 4(9.7)(9.3) = 663.16 > 0$, so there are solutions to this equation Hence there are values of T for which $F = 23$ / hence, the fact is not true in the biologist's model	M1 M1 A1 **[3]**	Sets 23 or 23000 equal to (b) Method to show their 3TQ has solutions, i.e. discriminant, completing the square, calculus, explicitly finding solutions, … Shows/explains that their 3TQ has solutions and gives a conclusion, i.e. ' hence, there are values of T for which $F = 23$', 'hence, fact is not true in biologist's model', 'qed', 'as required', …
5 (a)	$x\dfrac{dy}{dx} + y = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{y}{x}$ Hence, at P, we have $\dfrac{dy}{dx} = -\dfrac{1}{t^2}$ $y - 5t^{-1} = -\dfrac{1}{t^2}(x - 5t)$ $t^2 y - 5t = -x + 5t$ $\Rightarrow t^2 y + x = 10t$ **AG**	M1 A1 M1 A1 **[4]**	Method to find the derivative of the curve Correct gradient of the tangent at P Method to find equation of the line T_1 using the coordinates of P and their gradient at P Complete and convincing proof with no errors seen
5 (b)	$s^2 y + x = 10s$	B1 **[1]**	Correct equation in any form
5 (c)	Subtracting the two equations gives $t^2 y - s^2 y = 10t - 10s$ $\Rightarrow y = \dfrac{10(t-s)}{t^2 - s^2} = \dfrac{10(t-s)}{(t-s)(t+s)} = \dfrac{10}{t+s}$ $\Rightarrow x = 10s - s^2\left(\dfrac{10}{t+s}\right) = \dfrac{10st}{t+s}$	M1* A1 M1(dep*) A1 **[4]**	Eliminates one of the variables from the equation of T_1 and **their** T_2 Correct y coordinate in its simplest form Uses their y coordinate to find the x coordinate Correct x coordinate in its simplest form