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| Other Names | |
| Candidate Signature | |

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| Examiner Comments | Total Marks |
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# FURTHER MATHEMATICS

## AS LEVEL CORE PURE

CM

Bronze Set B (Edexcel Version)

Time allowed: 1 hour and 40 minutes

**Instructions to candidates:**

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer ALL of the questions.
- You must write your answer for each question in the spaces provided.
- You may use a calculator.

**Information to candidates:**

- Full marks may only be obtained for answers to ALL of the questions.
- The marks for individual questions and parts of the questions are shown in round brackets.
- There are 10 questions in this question paper. The total mark for this paper is 80.

**Advice to candidates:**

- You should ensure your answers to parts of the question are clearly labelled.
- You should show sufficient working to make your workings clear to the Examiner.
- Answers without working may not gain full credit.

AS/FM/CP



1 1 3 3 1 2 1 2 8 0 0 0 4

**1** Use standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} r(r-2)(r+2) = \frac{1}{4}n(n+1)(an^2+bn+c)$$

where $a$, $b$ and $c$ are integers to be found. **(4)**

**Question 1 continued**

**TOTAL 4 MARKS**

**2** The cubic equation

$$2z^3 - z^2 + z = 0$$

has three roots $\alpha, \beta$ and $\gamma$.

Without solving the equation, find the cubic equation whose roots are $(\alpha + 1)$, $(\beta + 1)$ and $(\gamma + 1)$, giving your answer in the form $w^3 + aw^2 + bw + c = 0$, where $a$, $b$ and $c$ are constants to be found. **(4)**

Do not write outside the box

**Question 2 continued**

**TOTAL 4 MARKS**

**3** Three planes are defined by the system of equations

$$4x + y + 2z = 4$$

$$kx - 2y + 3z = 10$$

$$y + (k - 1)z = 2$$

where $k$ is a constant.

(a) Find the values of $k$ for which the three planes do not intersect at a unique point. **(3)**

For both of the values of $k$ found in part (b),

(b) (i) show that the equations form an inconsistent system. **(3)**

(ii) Hence, identify the geometrical configuration of the planes in these cases. **(1)**

Do not write outside the box

**Question 3 continued**

**TOTAL 7 MARKS**

**4** The quadratic equation

$$3x^2 - kx + 14 = 0$$

has two positive real roots $\alpha$ and $\beta$ in the ratio 7:6.

Find the value of the constant $k$. **(5)**

*Do not write outside the box*

**Question 4 continued**

**TOTAL 5 MARKS**

**5** The complex numbers $z_1$ and $z_2$ are such that

$$z_1 = -8\sqrt{3} + 4\text{i}, \qquad z_2 = 7 + \text{i}\sqrt{3}$$

(a) Find

(i) $z_1 z_2$ **(2)**

(ii) $\dfrac{z_1}{z_2}$ **(3)**

giving your answers in the form $x + \text{i}y$.

On an Argand diagram with origin $O$, the complex numbers $z_1$ and $z_2$ are represented by the points $A$ and $B$ respectively.

(b) Show the points $A$ and $B$ on an Argand diagram. **(1)**

(c) Without using a calculator, prove that the angle $OAB$ is $\dfrac{5\pi}{6}$. **(2)**

*Do not write outside the box*

**Question 5 continued**

**TOTAL 8 MARKS**

**6** The $2 \times 2$ matrix **A** represents a reflection in the line $y = x$.

(a) Write down the matrix **A**. **(1)**

The transformation $T_1$ is a reflection in the line $y = x$ followed by an enlargement about the origin by a scale factor $k$, where $k \neq 0$ and $k \neq 1$. The $2 \times 2$ matrix **B** represents $T_1$.

(b) Find, in terms in $k$, the matrix **B**. **(3)**

The point $P$ is the only point invariant under $T_1$.

(c) Write down the coordinates of $P$. **(1)**

The transformation $T_2$ is represented by the matrix **C**, where

$$\mathbf{C} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$$

The transformation $T$ is the transformation $T_1$ followed by the transformation $T_2$.

The point $(1, 4)$ is mapped to the point $(20, 32)$ by the transformation $T$.

(d) Find the image of the point $(2, 3)$ under $T$. **(5)**

(e) Is the point $P$ also invariant under $T$? Justify your answer. **(2)**

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**Question 6 continued**

**Question 6 continued**

Do not write outside the box

**Question 6 continued**

**TOTAL 12 MARKS**

**7**

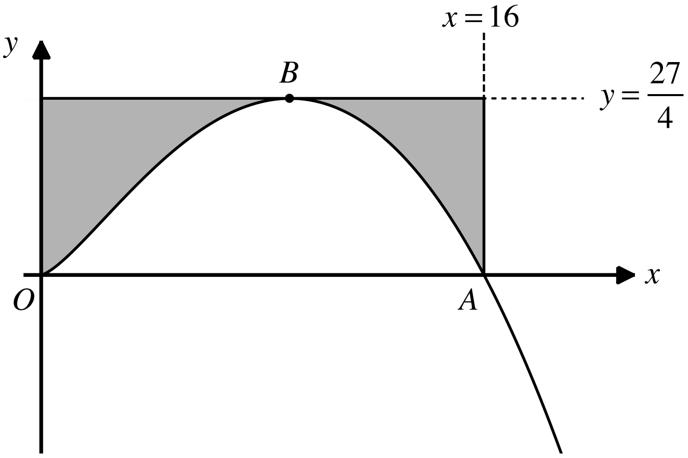


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \sqrt{x^3} - \frac{1}{4}x^2$$

The curve $C$ crosses the $x$-axis at the origin $O$ and the point $A(a, 0)$.

(a) Show that $a = 16$. **(2)**

The maximum point on the curve $C$ occurs at $B(b, c)$.

(b) Find the value of $b$ and hence show that $c = \frac{27}{4}$. **(4)**

The shaded region $R$, shown shaded in Figure 1, is bounded by the curve, the $y$-axis and the lines $y = \frac{27}{4}$ and $x = 16$. The region $R$ is rotated 360 degrees about the $x$-axis to form a solid with volume $V$.

(c) Use calculus to find $V$.
Give your answer to three significant figures. **(7)**

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**Question 7 continued**

**Question 7 continued**

*Do not write outside the box*

**Question 7 continued**

**TOTAL 13 MARKS**

**8** The lines $\mathbf{r}_1$ and $\mathbf{r}_2$ are defined such that

$$\mathbf{r}_1 = (\mathbf{i} + \mathbf{j}) + \lambda(-2\mathbf{i} + \mathbf{j} + 3\mathbf{z})$$

$$\mathbf{r}_2 = (-2\mathbf{i} + \mathbf{z}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{z})$$

(a) Show that the lines $\mathbf{r}_1$ and $\mathbf{r}_2$ are skew. **(4)**

(b) Show that the vector $(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ is normal to both $\mathbf{r}_1$ and $\mathbf{r}_2$. **(3)**

The plane $\Pi$ is equidistant from the lines $\mathbf{r}_1$ and $\mathbf{r}_2$.

(c) Find the equation of the plane $\Pi$.
Give your answer in the form $ax + by + cz = d$. **(3)**

Do not write outside the box

**Question 8 continued**

**TOTAL 10 MARKS**

**9** $$f(z) = z^3 - 13z^2 + pz + q$$

where $p$ and $q$ are constants.

The equation $f(z) = 0$ has roots $z_1$, $z_2$ and $z_3$.
When plotted on an Argand diagram, the points representing $z_1$, $z_2$ and $z_3$ lie on the circle $|z| = 5$.

Given that $z_1 = 5$, find the values of $p$ and $q$. **(8)**

Do not write outside the box

**Question 9 continued**

**TOTAL 8 MARKS**

**10** (a) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = n(n + 1)$$

is divisible by 2. **(4)**

(b) Hence, or otherwise, use induction to prove that for $n \in \mathbb{Z}^+$

$$g(n) = n^3 - n$$

is divisible by 6. **(5)**

**Question 10 continued**

**Question 10 continued**

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**Question 10 continued**

**END OF PAPER** **TOTAL 9 MARKS**

**TOTAL FOR PAPER IS 80 MARKS**