| Surname | |
|---|---|
| Other Names | |
| Candidate Signature | |

| Centre Number | | | | | | Candidate Number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

| Examiner Comments | Total Marks |
|---|---|
| | |

# FURTHER MATHEMATICS

## AS LEVEL CORE PURE

**CM**

Bronze Set A (Edexcel Version)

Time allowed: 1 hour and 40 minutes

**Instructions to candidates:**

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer ALL of the questions.
- You must write your answer for each question in the spaces provided.
- You may use a calculator.

**Information to candidates:**

- Full marks may only be obtained for answers to ALL of the questions.
- The marks for individual questions and parts of the questions are shown in round brackets.
- There are 10 questions in this question paper. The total mark for this paper is 80.

**Advice to candidates:**

- You should ensure your answers to parts of the question are clearly labelled.
- You should show sufficient working to make your workings clear to the Examiner.
- Answers without working may not gain full credit.

**1** The quadratic equation

$$2x^2 = 5x - 6$$

has roots $\alpha$ and $\beta$.

(a) Write down the value of $\alpha + \beta$ **and** the value of $\alpha\beta$. **(2)**

(b) Hence, find the value of $(\alpha + 2)(\beta + 2)$. **(2)**

The quadratic equation

$$4x^2 + ax + b = 0$$

where $a$ and $b$ are integers has roots $\alpha^2$ and $\beta^2$.

(c) Find the values of $a$ and $b$. **(4)**

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**Question 1 continued**

**TOTAL 8 MARKS**

**2** The complex numbers $z_1 = -4 - 3\text{i}$ and $z_2 = a + 7\text{i}$.

(a) Express $z_1$ in modulus-argument form.
Give the argument correct to three significant figures. **(3)**

(b) Find, in terms of $a$, the complex number $z_1z_2$, giving your answer in the form $c + \text{i}d$. **(2)**

(c) Given that $z_1z_2$ is real, find the value of $a$. **(2)**

Given instead that $z_1z_2$ lies on the imaginary axis on the Argand diagram,

(d) find the value of $a$. **(2)**

**Question 2 continued**

**TOTAL 9 MARKS**

**3** (a) Show that the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is perpendicular to **both** $3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$. **(3)**

The plane $\Pi$ contains the line $\dfrac{x-1}{3} = \dfrac{y+2}{-4} = z - 3$ and is parallel to $\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

(b) Find the Cartesian equation of the plane $\Pi$.

Give your answer in the form $ax + by + cz = d$. **(3)**

**Question 3 continued**

**TOTAL 6 MARKS**

**4** The cubic equation

$$x^3 - 9x^2 + 2x + 1 = 0$$

has roots $\alpha, \beta$ and $\gamma$.

Without solving the equation, find the cubic equation whose roots are $(\alpha + 1)$, $(\beta + 1)$ and $(\gamma + 1)$, giving your answer in the form $w^3 + pw^2 + qw + r = 0$, where $p$, $q$ and $r$ are integers to be found. **(5)**

**Question 4 continued**

**TOTAL 5 MARKS**

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**5** The curve $C$ has the equation $y = x^{\frac{3}{2}}$, $x > 0$.

The area bounded by the curve $C$, the $x$-axis and the lines $x = 1$ and $x = k$ is rotated $360^{\circ}$ about the $x$-axis to form a solid $S$.

Given that the solid $S$ has a volume $600\pi$, find the value of the constant $k$. **(5)**

**Question 5 continued**

**TOTAL 5 MARKS**

**6** The matrix **A** is defined such that

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(a) Find an expressions for $\mathbf{A}^2$, $\mathbf{A}^3$ and $\mathbf{A}^4$. **(3)**

(b) Hence, write down a general expression for $\mathbf{A}^k$, where $k$ is a positive integer. **(1)**

The matrix **B** is defined such that

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

(c) Prove, by induction, that for all $n \in \mathbb{Z}^+$,

$$\mathbf{B}^n = \begin{pmatrix} 1 & 1 & 2n-1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

**(5)**

**Question 6 continued**

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**Question 6 continued**

Do not write outside the box

**Question 6 continued**

**TOTAL 9 MARKS**

**7** (a) Using induction, prove that, for

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

**(5)**

(b) Hence, show that

$$\sum_{r=1}^{n}(4r^2-1) = \frac{1}{3}n(4n^2+6n-1)$$

**(3)**

(c) Use the result in (b) and showing all of your working, evaluate $\sum_{r=10}^{30}(4r^2-1)$. **(2)**

**Question 7 continued**

**Question 7 continued**

Do not write outside the box

**Question 7 continued**

**TOTAL 12 MARKS**

**8** The $2 \times 2$ matrix **A** represents a stretch of scale factor 5 parallel to the $x$-axis.

(a) Write down the matrix **A**. **(1)**

The matrix **B** is $\begin{pmatrix} 1 & 3 \\ 5 & k \end{pmatrix}$, where $k$ is a constant.

The transformation $T$ is the transformation represented by the matrix **A** followed by the tranformation represented by the matrix **B**.

The vertices of a triangle $T_1$ are mapped by $T$ to points that form the vertices of a triangle $T_2$.

Given that $T_1$ and $T_2$ have the same area,

(b) find the possible values of $k$. **(5)**

**Question 8 continued**

**TOTAL 6 MARKS**

**9**

$$f(z) = z^3 - 23z^2 + 299z - 2197$$

(a) Show that $(z - 13)$ is a factor of $f(z)$. **(2)**

The points representing the three roots of $f(z) = 0$ are plotted on an Argand diagram.

(b) Show that these points lie on the circle $|z| = k$, where $k$ is a constant to be determined. **(7)**

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**Question 9 continued**

**TOTAL 9 MARKS**

**10** Two planes $\Pi_1$ and $\Pi_2$ have the equations $\mathbf{r}\cdot(\mathbf{i}+2\mathbf{j}+\mathbf{k})=1$ and $\mathbf{r}\cdot(2\mathbf{i}+3\mathbf{j}-2\mathbf{k})=-2$ respectively.

(a) Find the acute angle between the planes $\Pi_1$ and $\Pi_2$. **(3)**

The point $A$ lies on the line of intersection of the planes $\Pi_1$ and $\Pi_2$ and has $z$-coordinate 2. The line $l$ passes through the point $A$ and is parallel to $5\mathbf{i}-\mathbf{j}+3\mathbf{k}$.

(b) Find the equation of the line $l$. **(3)**

(c) (i) Find the point on the line $l$ that is closest to the point $(4, 2, -1)$. **(5)**

(ii) Hence, calculate the shortest distance between the line $l$ and the point $(4, 2, -1)$. **(2)**

**Question 10 continued**

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**Question 10 continued**

Do not write outside the box

**Question 10 continued**

**END OF PAPER** **TOTAL 13 MARKS**

**TOTAL FOR PAPER IS 80 MARKS**