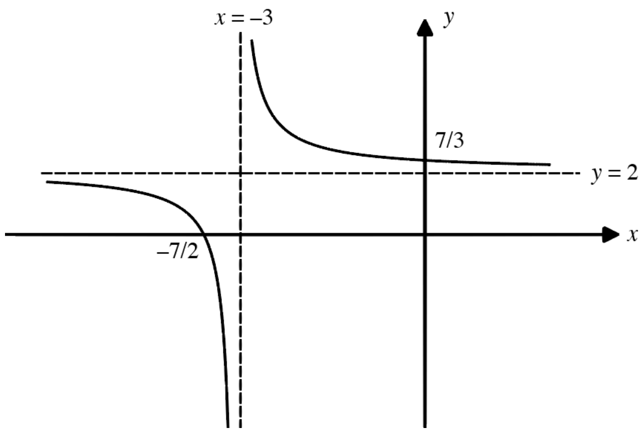
A Level
Maths

Bronze Set A, Paper 2 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1 (a)	$\left(-\frac{1}{2},0\right)$	B1 oe **[1]**	Correct coordinates oe. $x = -1/2$ alone is BOD B1
1 (b)	Asymptotes are the y–axis and the line $y = 2$	B1 B1 **[2]**	For asymptote at y–axis or $x = 0$ For asymptote at $y = 2$
1 (c)	$4 = 2+\frac{1}{x} \Rightarrow x=\frac{1}{2}$, so $B(1/2, 4)$ Then $\lvert AB\rvert = \sqrt{\left(-\frac{1}{2}-\frac{1}{2}\right)^2+(0-4)^2} = \sqrt{17}$	M1* M1(dep*) A1 oe **[3]**	Attempts to find x coordinate of B Method to find distance between A and B using their coordinates Correct distance oe (allow decimals)
1 (d)		B1 B1ft B1 B1ft **[4]**	Horizontal translation of the curve by ± 3 units Correct x intersection ft their (a) Correct y intersection Correct equations of asymptotes on graph or stated clearly. Asymptote at $x = -3$ must be correct but allow ft on their $y = 2$ from (b)
2 (a)	2.0886; 2.2578; 1.1367;	B1 B1 **[2]**	One value correctly given to four decimal places. All three values correctly given to four decimal places. Use of degrees is B0 B0

2 (b)	Area $\approx \frac{1}{2}(1)\left(0.9706+1.1367+2(2.0886+2.2578)\right)$ $= 5.40\ldots$	M1 A1ft A1 **[3]**	Uses correct formula for trapezium rule with their values from table Correct unsimplified expression for the area using their values Correct area. Awrt 5.4
3	$\dfrac{\cos^2\theta}{\sin\theta\tan\theta} \approx \dfrac{\left(1-\dfrac{\theta^2}{2}\right)^2}{\theta^2}$ $\approx \dfrac{1}{\theta^2}\left(1-\theta^2+\dfrac{\theta^4}{4}\right)$ $\approx \dfrac{1}{4}\theta^2+\theta^{-2}-1$ So $A = 1/4$, $B = 1$, $C = -1$	M1* M1(dep*) A1 **[3]**	Uses small angle approximations for sin, cos and tan Attempts to expand brackets and re-arrange into the correct form Obtains required expression convincingly or states values of A, B and C with sufficient working. ISW once correct form reached
4	$2(x + 3) \geq 4$, so $2x \geq -2$ So $x \geq -1$ $5x^2+9x-2<0 \Rightarrow (5x-1)(x+2)<0$ Critical values are 1/5 and –2 So solution to this inequality is $-2 < x < 1/5$ Hence range of values of x that satisfy both inequalities is $-1 \leq x < 1/5$	M1 A1 M1 A1 A1 A1ft **[6]**	Attempts to re-arrange inequality for x. Allow M1 for use of '=' sign and attempt to make x the subject Correct solution to linear inequality Method to find critical values of $5x^2 + 9x \pm 2$ Obtains both critical values Correct solution to quadratic inequality Correct solution to both inequalities. Allow the ft for using their solution to the linear and quadratic inequality. If their inequalities do not overlap, A1 is available for 'empty set' or 'no values of x satisfy both inequalities (simultaneously)'

5 (a)	$m=\frac{10-5}{7-3}=\frac{5}{4}$ So equation of $\mathbf{L}_1$ is $y-5=\frac{5}{4}(x-3)$ $\Rightarrow y=\frac{5}{4}x+\frac{5}{4}$	M1* M1(dep*) A1 **[3]**	Attempts to calculate gradient of the line Uses their gradient and a point to work out the equation of the line. If using $y = mx + c$, must see complete method to find c Correct equation of the line in the required form. Answer only is 3/3
5 (b)	Gradient of $\mathbf{L}_2$ is –2 Equation of $\mathbf{L}_2$ is $y - 2 = -2(x - 1)$ At A, $y = 0$, so $-2 = -2x + 2 \Rightarrow x = 2$ $\therefore A(2,0)$	B1 M1 A1 **[3]**	Correct gradient of $\mathbf{L}_2$, seen or implied Complete method to find the coordinates of A, i.e. working out equation of $\mathbf{L}_2$ and substituting in $y = 0$ Correct coordinates of A. $x = 2$ alone is A1 BOD.
5 (c)	$2k=10\Rightarrow k=5$	B1	**Parts (i), (ii) and (iii) should be marked together** Correct value of k. Seen or implied through correct gradient for $\mathbf{L}_3$
5 (c) (i)	(Gradient of $\mathbf{L}_1$ is 5/4 and gradient of $\mathbf{L}_2$ is –4), so neither parallel or perpendicular	B1ft	Correct conclusion ft their gradients
5 (c) (ii)	(Gradient of $\mathbf{L}_1$ is 5/4 and gradient of $\mathbf{L}_3$ is 5/4), so parallel	B1ft	Correct conclusion ft their gradients
5 (c) (iii)	(Gradient of $\mathbf{L}_2$ is –4 and gradient of $\mathbf{L}_3$ is 5/4), so they are neither parallel or perpendicular	B1ft **[4]**	Correct conclusion ft their gradients

6 (i)	Let $y = a^{kx}$ $\ln y = \ln(a^{kx}) \Rightarrow \ln y = kx \ln a$ Then $\frac{1}{y}\frac{dy}{dx} = k\ln a$ $\Rightarrow \frac{dy}{dx} = y(k\ln a) = ka^{kx}\ln a$ **AG**	B1 M1 A1 **[3]**	States or implies derivative of $\ln(y)$ as $\frac{1}{y}\frac{dy}{dx}$ Re-arranges to make dy/dx the subject and replaces y with a^{kx} Complete and convincing proof with no errors seen
6 (i) **ALT**	$a^{kx} = \mathrm{e}^{kx\ln a}$ So $\frac{d}{dx}(\mathrm{e}^{kx\ln a}) = k\ln a \times \mathrm{e}^{kx\ln a} = k\ln a \times a^{kx}$ $\Rightarrow \frac{d}{dx}(a^{kx}) = ka^{kx}\ln a$ **AG**	B1 M1 A1 **[3]**	Correctly writes a^{kx} in exponential form Method to differentiate $\mathrm{e}^{kx\ln(a)}$ (must have correct exponential form) Complete and convincing proof with no errors seen
6 (ii) (a)	$2x + 3x\frac{dy}{dx} + 3y - 4y\frac{dy}{dx} = 0$ $2x + 3x\frac{dy}{dx} + 3y - 4y\frac{dy}{dx} = 0$ $\frac{dy}{dx}(4y - 3x) = 2x + 3y$ $\Rightarrow \frac{dy}{dx} = \frac{2x+3y}{4y-3x}$ **AG**	B1 B1 M1 A1 **[4]**	States or implies derivative of $3xy$ as $3xy' + 3y$ States or implies derivative of $2y^2$ as $4yy'$ Equates to 0 and rearranges for dy/dx Complete and convincing proof with no errors seen

6 (ii) (b)	$\frac{2x+3y}{4y-3x}=1$ $\Rightarrow 2x+3y=4y-3x$ $\Rightarrow y=5x$ $x^2+3x(5x)-2(5x)^2+5=0$ $\Rightarrow x^2=\frac{5}{34}$ $\Rightarrow x=\pm\sqrt{\frac{5}{34}}$ So coordinates are $\left(\sqrt{\frac{5}{34}},5\sqrt{\frac{5}{34}}\right)$ and $\left(-\sqrt{\frac{5}{34}},-5\sqrt{\frac{5}{34}}\right)$	M1* A1 M1(dep*) A1 oe **[4]**	Sets derivative equal to 1 and obtains a linear relationship between y and x with one of the variables as the subject Obtains $y = 5x$ oe Substitutes their linear relation in the equation of the curve and solves for x Correct coordinates oe ISW
7	$\frac{3}{\tan x}+\frac{1-\tan x\tan\left(\frac{\pi}{4}\right)}{\tan x+\tan\left(\frac{\pi}{4}\right)}=4$ $\frac{3}{\tan x}+\frac{1-\tan x}{\tan x+1}=4$ $\frac{3\tan x+3+\tan x-\tan^2 x}{\tan^2 x+\tan x}=4$ $3\tan x+3+\tan x-\tan^2 x=4\tan^2 x+4\tan x$ $5\tan^2 x=3$ $\tan x=\pm\sqrt{\frac{3}{5}}$, so $x=\pm0.659\ldots$ $\therefore x=0.7,\ 2.5$	M1 A1 A1 oe A1 A1 **[5]**	Uses **correct** addition formula for $\tan(A+B)$ to obtain an equation solely in $\tan x$. Allow unsimplified $\tan(\pi/4)$ here. Correct unsimplified equation with $\tan(\pi/4) = 1$ used Obtains $5\tan^2 x = 3$ oe One value of x obtained correctly Both values of x obtained correctly and no others in range Must see consideration of ± for final A1

7 ALT	$\dfrac{3\sin x}{\cos x}+\dfrac{\sin x\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)\cos x}{\cos x\cos\left(\frac{\pi}{4}\right)-\sin x\sin\left(\frac{\pi}{4}\right)}=4$ $\dfrac{3\sin x}{\cos x}+\dfrac{\sin x+\cos x}{\cos x-\sin x}=5$ Rest is the same as the main scheme, but see notes	M1 A1 A1 oe A1 A1 **[5]**	Uses correct addition formula for $\sin(A+B)$ and $\cos(A+B)$ to obtain an equation in $\sin x$ and $\cos x$ only. Allow unsimplified $\sin(\pi/4)$ and $\cos(\pi/4)$ here Correct unsimplified equation with $\sin(\pi/4)$ and $\cos(\pi/4)$ substituted Obtains $\sin^2 x = 3/8$ or $\cos^2 x = 5/8$ oe One value of x obtained correctly Both values of x obtained correctly and no others in range Must see consideration of ± for final A1
8 (a)	$a_2=\dfrac{28}{5}$ $a_3=\dfrac{817}{140}$	M1 A1 A1 **[3]**	Substitutes 5 into recurrence relation Correct a_2 Correct a_3
8 (b)	$\sum_{n=1}^{3}(a_n-1)=(5-1)+\left(\dfrac{28}{5}-1\right)+\left(\dfrac{817}{140}-1\right)$ $=\dfrac{1881}{140}$	M1 M1 A1 **[3]**	For sight or use of $a_1+a_2+a_3$ For sight or use of -3 in sum [NB: Taking 1 from each of their a_ns and then adding them is M1 M1] Correct sum as an exact value. ISW once required form seen

8 (c) (i)	$a_{n+1} - a_n = \frac{1}{2}\left(a_n + \frac{6}{a_n} + 5\right) - a_n$ $= -\frac{1}{2}a_n + \frac{3}{a_n} + \frac{5}{2}$ $= \frac{-a_n^{\ 2} + 6 + 5a_n}{2a_n}$ $= -\frac{a_n^{\ 2} - 5a_n - 6}{2a_n}$ $= -\frac{(a_n - 6)(a_n + 1)}{2a_n}$ **AG**	M1 A1 **[2]**	Uses correct expression for the difference and then the collect terms and form a common denominator Complete and convincing proof with no errors seen
8 (c) (ii)	($a_n < 6 \Rightarrow a_n - 6 < 0$. Since the terms in the sequence are also positive, $-\frac{(a_n - 6)(a_n + 1)}{2a_n} > 0$.) (Hence,) $a_{n+1} - a_n > 0$ $(a_{n+1} > a_n)$, so the sequence is increasing	B1 **[1]**	States $a_{n+1} - a_n > 0$ and so the sequence is increasing. NB: the detailed justification in the scheme is not necessary and is for insight only.
9 (a)	$\overrightarrow{AB} = \begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} -5 \\ -5 \\ p-1 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 5 \\ 5 \\ p-1 \end{pmatrix}$	B1 B1 B1 **[3]**	One vector correct Two vectors correct All three vectors correct They must be vectors, so do not accept coordinates. SC B1: **all** three vectors correct but given as coordinates

9 (b)	$\lvert\overrightarrow{AB}\rvert = 10\sqrt{2}$, $\lvert\overrightarrow{AC}\rvert = \sqrt{50+(p-1)^2}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{50+(p-1)^2}$	M1 A1 A1 **[3]**	Method to find magnitude of one of their direction vectors One vector correct. Accept equivalent forms, e.g. √200 All three vectors correct. Accept equivalent forms
9 (c)	$\left(10\sqrt{2}\right)^2 = \left(\sqrt{50+(p-1)^2}\right)^2 + \left(\sqrt{50+(p-1)^2}\right)^2 - \ldots$ $\ldots - 2\left(\sqrt{50+(p-1)^2}\right)\left(\sqrt{50+(p-1)^2}\right)\cos 60$ $200 = 100 + 2(p-1)^2 - 50 - (p-1)^2$ $(p-1)^2 - 150 = 0$ $p^2 - 2p + 1 - 150 = 0$ $p^2 - 2p - 149 = 0$ **AG**	M1* M1(dep*) A1 **[3]**	Uses cosine rule with their (b) Attempts to simplify expression, correctly eliminating all radicals Convincingly obtains given result. Final answer with no errors
9 (c) ALT	$\cos 60 = \dfrac{\begin{pmatrix}-5\\-5\\p-1\end{pmatrix}\cdot\begin{pmatrix}5\\5\\p-1\end{pmatrix}}{\left(\sqrt{50+(p-1)^2}\right)\left(\sqrt{50+(p-1)^2}\right)}$ $\dfrac{1}{2} = \dfrac{-25-25+(p-1)^2}{50+(p-1)^2}$ $50+(p-1)^2 = -100 + 2(p-1)^2$ $(p-1)^2 - 150 = 0$ $p^2 - 2p + 1 - 150 = 0$ $p^2 - 2p - 149 = 0$ **AG**	M1* M1(dep*) A1 **[3]**	Uses dot product definition Computes dot product and re-arranges into the form of a 3TQ Convincingly obtains given result. Final answer with no errors
9 (d)	$p = 1 \pm 5\sqrt{6}$	B1 **[1]**	Correct values of p

9 (e)	$\frac{1}{2}\left(50+(1+5\sqrt{6}-1)^2\right)\sin(60)=50\sqrt{3}$	M1 A1 **[2]**	Use of ½ $ab\sin C$ with their magnitudes and largest p Correct area of triangle in exact form. ISW once exact form seen
9 (e) ALT	$\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=\frac{1}{2}\left\|\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -10 & -10 & 0\\ -5 & -5 & 5\sqrt{6}\end{vmatrix}\right\|$ $=\frac{1}{2}\sqrt{(-50\sqrt{6})^2+(50\sqrt{6})^2+0}$ $=\frac{1}{2}\sqrt{2}(50\sqrt{6})$ $=25\sqrt{12}$ $=50\sqrt{3}$	M1 A1 **[2]**	Complete method to find area of the triangle with their magnitudes and largest p. Need to see evaluation of the cross product **and** then candidates taking the magnitude of their result [NB: can use cross-product of **any** two of the direction vectors] Correct area of triangle in exact form. ISW once exact form seen
10 (a)	$\frac{\cos x-\sin 2x}{1-2\cos 2x}\equiv\frac{\cos x-2\cos x\sin x}{1-2(1-2\sin^2 x)}$ $\equiv\frac{\cos x(1-2\sin x)}{1-2+4\sin^2 x}$ $\equiv\frac{\cos x(1-2\sin x)}{4\sin^2 x-1}$ $\equiv\frac{\cos x(1-2\sin x)}{(2\sin x-1)(2\sin x+1)}$ $\equiv\frac{-\cos x}{1+2\sin x}$	M1 A1 M1 A1 **[4]**	Substitutes in correct formulae to obtain an expression only in $\sin(x)$ and $\cos(x)$ Correct unsimplified expression M1A0 is available if they lose the "1– " in the denominator Uses difference of two squares Complete and convincing proof with no errors seen Scalar multiples of the final answer should be accepted **ALTs:** 1. If working from RHS, M1 A1 for multiplying top and bottom by (1–2sin$x$) and M1 A1 for final answer. 2. If working from both sides, M1 A1 for using correct double angle formulae (as above) and M1 A1 for getting the sides to meet

10 (b)	$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{-\cos x}{1+2\sin x}dx = \left[-\frac{1}{2}\ln(1+2\sin x)\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= -\frac{1}{2}\ln\left(1+2\sin\frac{\pi}{2}\right) - -\frac{1}{2}\ln\left(1+2\sin\frac{\pi}{3}\right)$ $= -\frac{1}{2}\ln(3) + \frac{1}{2}\ln\left(1+\sqrt{3}\right)$ $= \frac{1}{2}\ln\left(\frac{1+\sqrt{3}}{3}\right)$	M1* A1 M1(dep*) A1 **[4]**	States integral of the form $a\ln(1+2\sin x)$ Correct integral Substitutes limits into integral in the correct order Correct answer. Allow equivalent expressions for the argument of the ln term, including decimals: e.g. ½ ln(0.91) is A1
11 (a)	$g'(x) = \frac{3}{(x+1)^2}$ $g'(x) = \frac{3}{(x+1)^2} > 0$ (on its domain), so g is an increasing function	B1* B1(dep*) **[2]**	Correct derivative in any form States that g' is positive and concludes that g is thus an increasing function.
11 (b)	$0 < g(x) < 3$	B1 B1 **[2]**	Correct lower limit of range Correct upper limit of range Accept equivalent notations, i.e. $g(x) \in (0,3)$. Accept use of y for $g(x)$, but not x. (0, 3) alone is B1 BOD. Allow $0 <$ range < 3. Use of $\le$ is B0.
11 (c)	Domain of fg is $(-\infty,-2)$ Range of fg is $2 < fg(x) < \sqrt{13}$	B1 B1ft **[2]**	Correct domain Correct range ft candidate's range of g. Notation is accepted in the same way as described in 11 (b).

11 (d) (i)	$\text{fg}(x)=\sqrt{\left(\frac{-3}{x+1}\right)^2+4}$	B1 **[1]**	**Parts (i) and (ii) can be marked together** Correct unsimplified expression for fg(x)
11 (d) (ii)	$\left(-\frac{3}{x+1}\right)^2+4=9$ $\left(-\frac{3}{x+1}\right)^2=5$ $\Rightarrow -\frac{3}{x+1}=\pm\sqrt{5}$ $x=-1\pm\frac{3}{\sqrt{5}}$ $\Rightarrow x=-1\pm\frac{3}{5}\sqrt{5}$ But domain of fg is $(-\infty,-2)$, so reject $x=-1+\frac{3}{5}\sqrt{5}$ and accept $x=-1-\frac{3}{5}\sqrt{5}$	M1* M1(dep*) A1 A1 **[4]**	Sets their fg(x) = 3 and squares both sides Attempts to solve the quadratic equation, i.e. by square rooting both sides. Must see generation of two solutions and use of ± Both solutions to the equation unsimplified in any form Explicitly rejects the solution outside of the domain and gives the single solution in the requested form. The rejection must be explicit but no reason needs to be given. However, if the wrong reason is given for rejection, then A0 [Only correct solution given with **explicit** rejection is SC A2]
12 (a)	$\int t^2\text{e}^{-2t}\,dt=-\frac{1}{2}t^2\text{e}^{-2t}+\frac{1}{2}\int 2t\text{e}^{-2t}\,dt$ $=-\frac{1}{2}t^2\text{e}^{-2t}+\left[-\frac{1}{2}t\text{e}^{-2t}+\frac{1}{2}\int \text{e}^{-2t}\,dt\right]$ $=-\frac{1}{2}t^2\text{e}^{-2t}-\frac{1}{2}t\text{e}^{-2t}-\frac{1}{4}\text{e}^{-2t}\quad(+k)$ So $\int_0^1 t^2\text{e}^{-2t}\,dt=\left[-\frac{1}{2}\text{e}^{-2(1)}-\frac{1}{2}\text{e}^{-2(1)}-\frac{1}{4}\text{e}^{-2(1)}\right]-\left[0+0-\frac{1}{4}\right]$ $=\frac{1}{4}-\frac{5}{4}\text{e}^{-2}$	M1* A1 A1 M1(dep*) A1 **[5]**	Uses IBP once and obtains expression of the form $at^2\text{e}^{-2t}+b\int \text{e}^{-2t}\,dt$ Correct expression after applying IBP once Correct unsimplified expression after applying IBP twice Substitutes limits into their integral in the correct order Correct answer in terms of e or equivalent. Final answer.

12 (b)	$u = x+1 \Rightarrow du = dx$ Then $\int x\sqrt{x+1}\,dx = \int (u-1)\sqrt{u}\,du$ $= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du = \frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3} + k$ $= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + k$	B1 M1* M1(dep*) A1 A1ft **[5]**	Correct relation between *du* and *dx* Correct substitution for *u* in the integral (ignore what they do with *dx*) Correct method to evaluate their integral in *u* Correct integration in *u* (allow omission of a constant) Replaces *u* with *x* to obtain final integral in terms of *x* which includes a constant of integration ft their integral in *u*