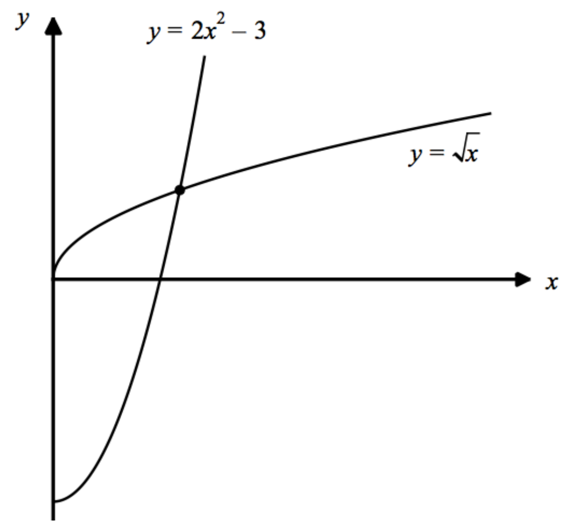
A Level
Maths

Bronze Set B, Paper 1 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1	$\frac{dy}{dx} = 2x - x^{-2} - 3e^{3x}$	M1 M1 A1 A1 oe **[4]**	Method to differentiate one of the x^n terms, $n \neq 0$ Uses the chain rule to differentiate the exponential term Any two terms differentiated correctly (unsimplified or better) All four terms differentiated correctly and simplified. Accept equivalent forms e.g. $\frac{1}{x^2}$ instead of x^{-2}
2 (a)	$14 = 3p + 2 \Rightarrow p = 4$	M1 A1 **[2]**	Sets up a correct equation using information about x_1 and x_2 Obtains the correct value of p
2 (b)	$x_3 = 4(14) + 2 = 58$ $x_4 = 4(58) + 2 = 234$ $\sum_{n=1}^{4} x_n = x_1 + x_2 + x_3 + x_4$ $= 3 + 14 + 58 + 234$ $= 309$	B1ft B1ft M1 A1 **[4]**	Correct value of x_3 ft their p Correct value of x_4 ft their p and their x_3 Complete method to find the sum Correct value of the sum
3 (a)	(Since θ is acute,) $\cos\theta = \sqrt{1 - \sin^2\theta}$ $= \sqrt{1 - p^2}$	M1 A1 **[2]**	Complete method to find $\cos\theta$ (allow $\pm$ here) Award method mark for complete method using right-triangle Correct expression of $\cos\theta$

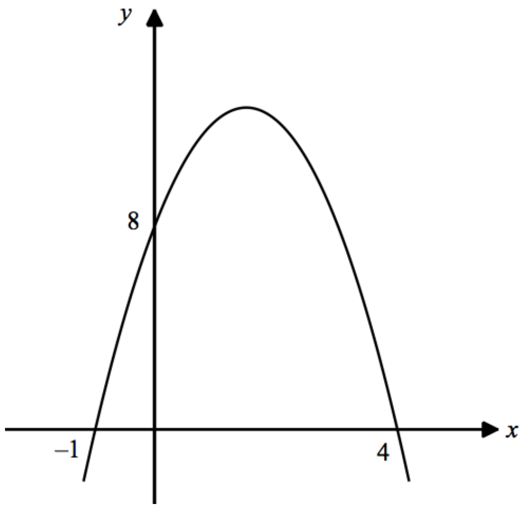
3 (b)	$\operatorname{cosec}2\theta=\frac{1}{\sin 2\theta}=\frac{1}{2\sin\theta\cos\theta}=\frac{1}{2p\sqrt{1-p^2}}$	M1 A1ft **[2]**	Complete method to find cosec2θ using their (a) Correct expression of cosec2θ
3 (c)	$\sin(\theta-45)=\sin\theta\cos45-\cos\theta\sin45$ $=\frac{1}{\sqrt{2}}\left(p-\sqrt{1-p^2}\right)$	M1 A1 **[2]**	Complete method to find $\sin(\theta-45)$ using their (a) **and** replacing some value for cos(45) and sin(45) Correct expression of $\sin(\theta-45)$
4	$2x+3y=-2\Rightarrow y=-\frac{2}{3}x-\frac{2}{3}$, so gradient of l_1 is $-\frac{2}{3}$ $\frac{-1-2}{-2-0}=\frac{3}{2}$, so gradient of l_2 is $\frac{3}{2}$ Since the product of the gradients $-\frac{2}{3}\times\frac{3}{2}=-1$, the lines l_1 and l_2 are perpendicular	B1 M1 A1 A1ft **[4]**	Correct gradient of l_1 seen or implied Attempts to find the gradient of l_2 (allow sign errors) Correct gradient of l_2 Correct conclusion ft their gradients giving correct and clear reasoning
5 (a)	$\mathrm{f}(x)>3$	B1 **[1]**	Correct range of f Condone y in place of f(x) but **do not accept** x
5 (b)	$\mathrm{fg}(x)=2\mathrm{e}^{\ln(4x)}+3=2(4x)+3=8x+3$ Hence, $\mathrm{fg}(x)=5\Rightarrow 8x+3=5$ $\Rightarrow x=\frac{1}{4}$	B1 M1 A1 **[3]**	Uses $\mathrm{e}^{\ln(4x)}=4x$ at any stage Finds fg(x) in any form and sets it equal to 5 Correct value of x
5 (c)	$y=2\mathrm{e}^x+3\Rightarrow \mathrm{e}^x=\frac{y-3}{2}$ $\Rightarrow x=\ln\left(\frac{y-3}{2}\right)$, so $\mathrm{f}^{-1}(x)=\ln\left(\frac{x-3}{2}\right)$ for $x>3$	M1 A1 B1ft **[3]**	Sets y equal to x and attempts to re-arrange for x (getting up to $\mathrm{e}^x=\ldots$ is OK) Correct expression for $\mathrm{f}^{-1}(x)$ Correct domain ft their 5(a)

6 (a)	$f(1)=\sqrt{1}-2(1)^2+3=2\ (>0)$ $f(2)=\sqrt{2}-2(2)^2+3=-3.587...(<0)$ since there has been a **change of sign** and **f is continuous** (on [1, 2]), f has a root between [1,2]	M1 A1 **[2]**	Calculates values of f(1) and f(2) Correctly calculated values and conclusion to complete the proof
6 (b)	$f(1.5)=-0.2752...$ $f'(x)=\frac{1}{2\sqrt{x}}-4x$, so $f'(1.5)=\frac{1}{2\sqrt{1.5}}-4(1.5)=-5.5917...$ Applying NR process: $\alpha_2=\alpha_1-\frac{f(\alpha_1)}{f'(\alpha_1)}$ $=1.5-\frac{(-0.2752...)}{(-5.5917...)}$ $=1.4507...$ so the second approximation is 1.45 to 3 sf	B1 M1* A1 M1(dep*) A1 **[5]**	Correct value of f(1.5) seen or implied anywhere Complete method to find $f'(1.5)$ Correct value of $f'(1.5)$ Valid attempt at Newton-Raphson using their values Correct second approximation to 3 sf
6 (c)	 since the graphs only intersect once, f(*x*) = 0 only has one root	M1 A1 **[2]**	Attempts to draw the graphs of $y=\sqrt{x}$ and $y=2x^2-3$ on the same axis Correctly drawn graphs and conclusion that states that the graphs intersect once and so there is only one root **Alternative:** M1 – draws the graph of $y = f(x)$ A1 – states that the graph only intersects the x-axis once and so the equation $f(x) = 0$ only has one root

7 (a)	$2 = A(x+1) + B(2-x)$ When $x = -1,\ 2 = 3B \Rightarrow B = \frac{2}{3}$ When $x = 2,\ 2 = 3A \Rightarrow A = \frac{2}{3}$ So $f(x) = \frac{2}{3(2-x)} + \frac{2}{3(x+1)}$	M1 A1 A1 **[3]**	Complete and correct method to find one of the values Correct value of A seen or implied by final answer Correct value of B seen or implied by final answer (values of A and B are enough, i.e. final line of ms not required)
7 (b)	$f(x) = \frac{2}{3}(2-x)^{-1} + \frac{2}{3}(x+1)^{-1}$ so $f'(x) = -\frac{2}{3}(2-x)^{-2}(-1) + \frac{2}{3}(x+1)^{-2}(1)$ $\Rightarrow f'(x) = \frac{2}{3(2-x)^2} - \frac{2}{3(x+1)^2}$ At minimum point $f'(x) = 0 \Rightarrow \frac{1}{(2-x)^2} = \frac{1}{(x+1)^2}$ Cross-multiplying gives $x^2 + 2x + 1 = 4 - 4x + x^2$ $6x = 3 \Rightarrow x = \frac{1}{2}$ $f\left(\frac{1}{2}\right) = \frac{8}{9}$ so minimum point has coordinates $\left(\frac{1}{2}, \frac{8}{9}\right)$	M1 A1 M1* M1(dep*) A1 **[5]**	Correct method to differentiate one of the terms, including intended use of the chain rule seen (this can be implied) Correct derivative Sets their derivative = 0 Carries out a complete process to solve their equation for x If they square root both sides, allow the M1 for square rooting but must see consideration of ± for the final A1 (otherwise A0^) Correct coordinates

7 (c)	$\int_0^{\frac{1}{2}} \mathrm{f}(x)\,dx = \int_0^{\frac{1}{2}} \left(\frac{2}{3(2-x)} + \frac{2}{3(x+1)}\right) dx$ $= \left[-\frac{2}{3}\ln(2-x) + \frac{2}{3}\ln(x+1)\right]_0^{\frac{1}{2}}$ $= \left(-\frac{2}{3}\ln\left(\frac{3}{2}\right) + \frac{2}{3}\ln\left(\frac{3}{2}\right)\right) - \left(-\frac{2}{3}\ln(2) + \frac{2}{3}\ln(1)\right)$ $= \frac{2}{3}\ln 2$	M1* A1ft M1(dep*) A1 **[4]**	States integral of the form $a\ln(2–x) + b\ln(x + 1)$ Correct indefinite integration ft their a and b Substitutes in the correct limits (ft their b) in the correct order Their upper limit from (b) must make sense with respect to the picture and function, i.e. be positive and less than 3/2 Obtains correct result in the correct form ISW
8	$\frac{\mathrm{d}}{\mathrm{d}\theta}(\sin\theta) = \lim_{h\to 0}\frac{\sin(\theta+h)-\sin\theta}{h}$ $= \lim_{h\to 0}\frac{\sin\theta\cos h + \sin h\cos\theta - \sin\theta}{h}$ $= \lim_{h\to 0}\left(\frac{\sin\theta(\cos h-1)}{h} + \frac{\cos\theta\sin h}{h}\right)$ $= (\sin\theta)\lim_{h\to 0}\frac{\cos h-1}{h} + (\cos\theta)\lim_{h\to 0}\frac{\sin h}{h}$ $= (\sin\theta)(0) + (\cos\theta)(1)$ $= \cos\theta$ **AG**	B1* M1* A1 M1(dep*) A1 **[5]**	Correct expression for the derivative Expands the compound angle (allow a sign error in the formula) Correct expression Groups the $\sin\theta$ terms and the $\cos\theta$ terms and attempts to apply the limit Complete and convincing proof with no errors seen **and** correct limiting process seen No consideration of limits is A0

9 (a)	p and q are odd numbers Let $p = 2n + 1$ and $q = 2m + 1$, where n and m are (positive) integers Then $p^2 + q^2 = (2n+1)^2 + (2m+1)^2$ $= 4n^2 + 4n + 1 + 4m^2 + 4m + 1$ $= 2(2n^2 + 2m^2 + 2n + 2n + 1)$ which is even. So the statement is true	B1 M1 A1 **[3]**	Deduces that p and q must both be odd Attempts to characterise p and q as potentially **distinct** odd integers and find the sum of the squares Complete, convincing and technical proof (need to see p and q explicitly defined and n, m defined as integers) with no errors and conclusion **Note 1:** if you don't see $p = 2n + 1$ and $q = 2m + 1$, but you do see $(2n + 1)^2 + (2m + 1)^2$ then give the M1 by implication (but the A1 is withheld unless characterisation is clear) **Note 2:** if p and q are given the same characterisation or n is used for both (for example), then M0 A0 **Note 3:** unproven statements such as 'sum of odd number is odd' are not good enough for the M1
9 (b)	Suppose for a contradiction that there are a finite number of primes. Let $p_1, p_2, \ldots, p_k$ be a collection of all the primes. Consider the number $P = p_1 p_2 \ldots p_k + 1$ (P must have a prime factor but) none of the primes $p_1, p_2, \ldots, p_k$ divide P, so P must be prime	M1* M1(dep*) A1 **[3]**	Attempts a proof by contradiction, assuming that there are finitely many primes Constructs the number P Complete and convincing proof with clear reasoning for why the construction of P implies the existence of another prime
9 (b) ALT	Suppose for a contradiction that there are a finite number of primes. Then the largest prime exists and let p_k be this prime Consider $P = p_k! + 1$. (P must have a prime factor but) none of p_k or any of the smaller primes divide P, so P must be prime	M1* M1(dep*) A1 **[3]**	Attempts a proof by contradiction, assuming that there are finitely many primes Constructs the number P Complete and convincing proof with clear reasoning for why the construction of P implies the existence of another prime

10 (a) (i)	$y = -2(x^2 - 3x) + 8$ $= -2\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 8$ $= -2\left(x - \frac{3}{2}\right)^2 + \frac{9}{2} + 8$ $= -2\left(x - \frac{3}{2}\right)^2 + \frac{25}{2}$ so $a = -2$, $b = -\frac{3}{2}$ and $c = \frac{25}{2}$	M1 A1 A1 **[3]**	Extracts a factor of –2 and attempts to complete the square on their left-over expression Correct unsimplified expression Completes the square correctly, obtaining the answer in the required form or values of *a*, *b* and *c* stated
10 (a) (ii)	Coordinates of the maximum point is $\left(\frac{3}{2}, \frac{25}{2}\right)$	B1 B1ft **[2]**	Maximum point Correct coordinates ft their 10(a)
10 (b)		B1 B1 B1 **[3]**	Correct shape of the graph Correct *x* intersections Correct *y* intersection
10 (c)	$-2x^2 + 6x + 8 = k(x + 2) \Rightarrow -2x^2 + 6x + 8 = kx + 2k$ $\Rightarrow 2x^2 + kx - 6x + 2k - 8 = 0$ $\Rightarrow 2x^2 + (k - 6)x + (2k - 8) = 0$ **AG**	M1 A1 **[2]**	Eliminates *y* from the two equations and attempts to move all the terms to one side Complete and convincing proof with no errors seen

10 (d)	If the curve and line are tangent, then they only intersect once, so $(k-6)^2-4(2)(2k-8)=0$ $\Rightarrow k^2-12k+36-8(2k-8)=0$ $\Rightarrow k^2-28k+100=0$ $k=\dfrac{-(-28)\pm\sqrt{(-28)^2-4(1)(100)}}{2(1)}$ $\Rightarrow k=14\pm4\sqrt{6}$	M1* A1 M1(dep*) A1 **[4]**	Sets discriminant of the equation equal to 0 Obtains the correct 3TQ Complete method to solve their quadratic for k Use of a calculator does not score the method mark if it leads to the wrong answer Correct values of k
11 (a)	$6=a+d$ $-7=a+6d$ $\Rightarrow 13=-5d$ $\Rightarrow d=-\dfrac{13}{5}$ $a=6-d=6+\dfrac{13}{5}$, so $a=\dfrac{43}{5}$	B1 B1 M1 A1 A1 **[5]**	One equation correct A second correct equation Attempts to solve their simultaneous equations Correct value of a or d Correct value of a and d
11 (b)	First term = 1.05, common difference = 0.10 In the nth week, she donates $150\times0.025=£3.75$ Hence $3.75=1.05+(n-1)(0.10)\Rightarrow n=28$ So sum of donations over the n week period is $S_{28}=\dfrac{28}{2}\left[2(1.05)+(28-1)(0.10)\right]=£67.20$	B1 B1 M1 A1 M1 A1 **[6]**	Correct first term and common difference of sequence seen or implied at any stage Correct amount of donation in the nth week seen or implied at any stage Sets up correct equation and attempts to find n Correct value of n Uses their values of a and d and their n to find S_n Correct sum of donations with units (accept 6720p)

12	**SUBSTITUTION (R1)**	M1	Chooses to use substitution. This is an overall process mark. Award for: 1) attempting use substitution $u = \ldots$, changing terms to u's 2) integrating and using appropriate limits
	Let $u = 2 - x$, then $x = 2 - u$ and $dx = -du$	B1	States substitution $u = 2 - x$ and a correct dx in terms of du (or equivalent)
	When $x = 1$, $u = 1$ and when $x = -1$, $u = 3$		
	So $\int_{-1}^{1} 5x\sqrt{2-x}\,dx = \int_{3}^{1} 5(2-u)\sqrt{u}(-du)$	M1*	Attempts to get **all** aspects of the integral in terms of u's Condone slips in signs and coefficients
	$= \int_{3}^{1} (-10\sqrt{u} + 5u\sqrt{u})\,du$		
	$= \left[-10u^{\frac{3}{2}}\left(\frac{2}{3}\right) + 5u^{\frac{5}{2}}\left(\frac{2}{5}\right)\right]_3^1$	M1**(dep*) A1	States or implies integral of the form $au^{3/2} + bu^{5/2}$ Correct integral
	$= -\frac{20}{3}(1)^{\frac{3}{2}} + 2(1)^{\frac{5}{2}} + \frac{20}{3}(3)^{\frac{3}{2}} - 2(3)^{\frac{5}{2}}$	M1(dep**)	Substitutes the **correct** limits into the integral in the correct order
	$= -\frac{20}{3} + 2 + \frac{20}{3}\sqrt{27} - 2\sqrt{243}$		
	$= -\frac{14}{3} + 20\sqrt{3} - 18\sqrt{3}$		
	$= 2\sqrt{3} - \frac{14}{3}$		
	$= \frac{1}{3}(6\sqrt{3} - 14)$ **AG**	A1 **[7]**	Obtains the given result convincingly with no errors seen

12 ALT	**SUBSTITUTION (R2)**	M1	Chooses to use substitution. This is an overall process mark. Award for: 1) attempting use substitution $u = \ldots$, changing terms to u's 2) integrating and using appropriate limits
	Let $u^2 = 2 - x$, then $2u\,du = -\,dx$, so $dx = -2u\,du$	B1	States substitution $u^2 = 2 - x$ and a correct dx in terms of du (or equivalent)
	When $x = 1$, $u = 1$ and when $x = -1$, $u = \sqrt{3}$ So $\int_{-1}^{1} 5x\sqrt{2-x}\,dx = \int_{\sqrt{3}}^{1} 5(2-u^2)u(-2u\,du)$ $= \int_{\sqrt{3}}^{1} (-20u^2 + 10u^4)du$	M1*	Attempts to get **all** aspects of the integral in terms of u's Condone slips in signs and coefficients
	$= \left[-\frac{20}{3}u^3 + 2u^5\right]_{\sqrt{3}}^{1}$	M1**(dep*) A1	States or implies integral of the form $au^3 + bu^5$ Correct integral
	$= -\frac{20}{3}(1)^3 + 2(1)^5 + \frac{20}{3}(\sqrt{3})^3 - 2(\sqrt{3})^5$ $= -\frac{14}{3} + 20\sqrt{3} - 2(9)\sqrt{3}$ $= 2\sqrt{3} - \frac{14}{3}$	M1(dep**)	Substitutes the **correct** limits into the integral in the correct order
	$= \frac{1}{3}(6\sqrt{3} - 14)$	A1 **[7]**	Obtains the given result convincingly with no errors seen

12 ALT	**PARTS (R3)** $\int_{-1}^{1} 5x\sqrt{2-x}\,dx = \left[\frac{5x(2-x)^{\frac{3}{2}}(2)}{3(-1)}\right]_{-1}^{1} + \frac{2}{3}\int_{-1}^{1} 5(2-x)^{\frac{3}{2}}dx$ $= \left[-\frac{10}{3}x(2-x)^{\frac{3}{2}}\right]_{-1}^{1} + \frac{2}{3}\left[\frac{5(2-x)^{\frac{5}{2}}(2)}{5(-1)}\right]_{-1}^{1}$ $= \left[-\frac{10}{3}x(2-x)^{\frac{3}{2}}\right]_{-1}^{1} + \frac{2}{3}\left[-2(2-x)^{\frac{5}{2}}\right]_{-1}^{1}$ $= -\frac{10}{3}(1)(1)^{\frac{3}{2}} + \frac{2}{3}(-2)(1)^{\frac{5}{2}} + \frac{10}{3}(-1)(3)^{\frac{3}{2}} + \frac{2}{3}(2)(3)^{\frac{5}{2}}$ $= -\frac{10}{3} - \frac{4}{3} - \frac{10}{3}\sqrt{27} + \frac{4}{3}\sqrt{243}$ $= -\frac{14}{3} - 10\sqrt{3} + 12\sqrt{3}$ $= 2\sqrt{3} - \frac{14}{3}$ $= \frac{1}{3}(6\sqrt{3} - 14)$ **AG**	M1 B1 M1* M1**(dep*) A1 M1(dep**) A1 **[7]**	Chooses to use parts. This is an overall process mark. Award for: 1) Attempting to use parts the correct way around 2) using limits States or implies $\int \sqrt{2-x}\,dx = -\frac{2}{3}(2-x)^{\frac{3}{2}}$ Uses parts correctly once and obtains expression of the form $Ax(2-x)^{\frac{3}{2}} + B\int(2-x)^{\frac{3}{2}}dx$ Integrates a second time to obtain integral of the form $Px(2-x)^{\frac{3}{2}} + Q(2-x)^{\frac{5}{2}}$ Correct integral of $-\frac{10}{3}x(2-x)^{\frac{3}{2}} - \frac{4}{3}(2-x)^{\frac{5}{2}}$ seen or implied This may be partitioned as in the mark scheme so you may see the integral in separate parts (the partitioned parts may be evaluated separately also) Substitutes correct limits into their integral in the correct order Obtains the given result convincingly with no errors seen

13 (a)	e.g. The liquid's surface is a circle of radius 1 m and so has area π	B1 **[1]**	Any sensible explanation/illustration of why $A = \pi$
13 (b)	$\pi \frac{dh}{dt} = -0.016\pi\sqrt{h}$ $\Rightarrow \frac{1}{\sqrt{h}}\frac{dh}{dt} = -0.016$ $\Rightarrow \int \frac{1}{\sqrt{h}} dh = \int -0.016\, dt$ $\Rightarrow 2\sqrt{h} = -0.016t + c$ When $t = 0, h = 4$, so $c = 2\sqrt{4} = 4$ $\Rightarrow 2\sqrt{h} = 4 - 0.016t$ $\Rightarrow \sqrt{h} = 2 - 0.008t$ $\Rightarrow h = (2 - 0.008t)^2$ **AG**	B1* B1(dep*) M1 A1 **[4]**	Separates variables correctly Attempted integration of one of the sides Uses initial conditions to find their c in an expression which contains the terms at and $b\sqrt{h},\ a, b \neq 0$ Obtains the given result convincingly with no errors seen
13 (c)	Tank is empty when $h = 0$, i.e. $(2 - 0.008t)^2 = 0$ $\Rightarrow 2 - 0.008t = 0$ $\Rightarrow t = 250$ s so 4.17 minutes	M1 A1 A1 **[3]**	Sets $h = 0$ and attempts to re-arrange for t If they expand into a 3TQ, must see a valid attempt to solve this Correct value of t in seconds Correct value of t in minutes
13 (d)	e.g. The area of the liquid's surface now changes as the liquid drains	B1 **[1]**	Correct explanation