Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				

Examiner Comments	

Total Marks

FURTHER MATHEMATICS

A LEVEL CORE PURE 2

CM

Bronze Set A (Edexcel Version)

Time allowed: 1 hour and 30 minutes

Instructions to candidates:

• In the boxes above, write your centre number, candidate number, your surname, other names and signature.

• Answer ALL of the questions.

• You must write your answer for each question in the spaces provided.

• You may use a calculator.

Information to candidates:

• Full marks may only be obtained for answers to ALL of the questions.

• The marks for individual questions and parts of the questions are shown in round brackets.

• There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to candidates:

• You should ensure your answers to parts of the question are clearly labelled.

• You should show sufficient working to make your workings clear to the Examiner.

• Answers without working may not gain full credit.

1 2 3 3 1 3 2 1 8 0 0 0 4

1 Given that $y = \arcsin(4x)$, prove that

$$\frac{dy}{dx} = \frac{4}{\sqrt{1-16x^2}}$$

(3)

Question 1 continued

TOTAL 3 MARKS

2 The quadratic equation

$$4x^2 + 9x - 1 = 0$$

has roots α and β.

(a) Without solving the equation, find the value of

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

(5)

(b) Find the equation with roots $(4\alpha - 1)$ and $(4\beta - 1)$.
Give your answer in the form $w^2 + aw + b = 0$, where a and b are constants. **(3)**

Question 2 continued

TOTAL 8 MARKS

3 (a) Write down the exponential definition of $\tanh(x)$. **(1)**

(b) Prove, using the exponential definition of $\tanh(x)$, that

$$\frac{\mathrm{d}}{\mathrm{d}x}[\tanh(x)] = 1 - \tanh^2(x)$$

(3)

The function f is defined such that

$$\mathrm{f}(x) = \tanh(x) - 3\ln(x + 1) + 4x + 3, \quad x > -1$$

(c) Find the first three terms in the Maclaurin series of $\mathrm{f}(x)$.
Show your working clearly and give each term in its simplest form. **(5)**

Question 3 continued

TOTAL 9 MARKS

4

$$\mathbf{A} = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

The 3×3 matrix **A** represents a rotation about the y-axis by an angle θ.

(a) (i) Explain geometrically why $\det(\mathbf{A}) = +1$. **(2)**

(ii) Verify, by calculation, that $\det(\mathbf{A}) = +1$. **(3)**

The transformation T is a reflection about the y-axis followed by a rotation about the y-axis by an angle of 30°. The 3×3 matrix **M** represents T.

The point P has the coordinates $(2, 1, -4)$. The point P is mapped under T to the point Q.

(b) Find the coordinates of Q. **(4)**

The line l_1 passes through the points P and Q.

The line l_2 has the equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$.

(c) Show that the lines l_1 and l_2 are skew. **(6)**

Question 4 continued

Question 4 continued

Question 4 continued

TOTAL 15 MARKS

5 The complex numbers z_1 and z_2 are given by

$$z_1 = 5 + 2\mathrm{i}, \qquad z_2 = -1 - 13\mathrm{i}$$

The complex number $z = p + \mathrm{i}q$, where p and q are constants.

Given that $\dfrac{z + 2z_1}{4z - z_2} = 1$,

(a) show that $p = 3$ and $q = -3$. **(4)**

(b) Hence, express z in the form $r(\cos\theta + \mathrm{i}\sin\theta)$. **(3)**

(c) Find the complex numbers w that satisfy $w^4 = z$. **(4)**

The complex numbers w are plotted on an Argand diagram.

The complex numbers w lie on the circle $|w| = a$.

(d) Write down the value of a. **(1)**

Question 5 continued

Question 5 continued

Question 5 continued

TOTAL 12 MARKS

6 (a) Using a suitable substitution, show that

$$\int \frac{1}{\sqrt{9-x^2}}\, dx = \sin^{-1}\left(\frac{x}{3}\right) + C$$

where C is a constant. **(3)**

(b) Find the particular solution to the differential equation

$$t\frac{dx}{dt} + x = \frac{1}{\sqrt{9-t^2}}, \quad |t| < 3,\ t \neq 0$$

given that when $t = \frac{1}{2}$, $x = 0$. **(5)**

Question 6 continued

TOTAL 8 MARKS

7

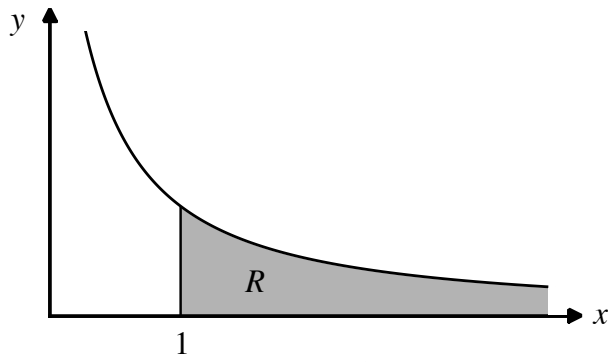

Figure 1

Figure 1 above shows a sketch of the curve C which has equation $y = f(x)$, where

$$f(x) = \sqrt{\frac{x+1}{x^3}}, \quad x > 0$$

The infinite region R, shown shaded in Figure 1, is bounded by the curve C, the x-axis and the line = 1. The region R is rotated by 2π radians about the x-axis to form the solid S, which has volume V.

(a) Show that

$$V = \pi \int_1^\infty \left(\frac{1}{x^2} + \frac{1}{x^3} \right) dx$$

(2)

(b) Explain why $\int_1^\infty \left(\frac{1}{x^2} + \frac{1}{x^3} \right) dx$ is an improper integral. **(1)**

(c) Show that V is finite and find its value. **(4)**

Question 7 continued

TOTAL 7 MARKS

8

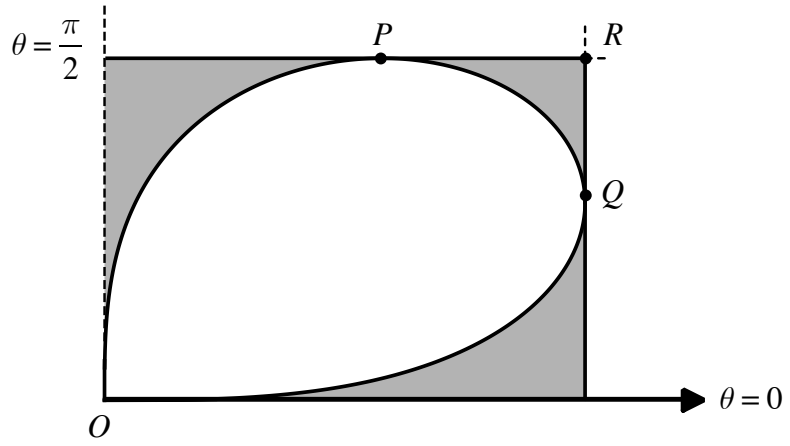

Figure 2

Figure 2 shows a sketch of the polar curve C with equation

$$r = a\sqrt{\sin(2\theta)}, \qquad 0 \leq \theta \leq \frac{\pi}{2}$$

The point P is the point on C where the tangent to C is parallel to the initial line and the point Q is the point on C where the tangent is perpendicular to the initial line. The two tangents meet at the point R and the point O is the pole.

(a) Show that the point P lies on the line $\theta = \frac{\pi}{3}$. **(6)**

The point Q lies on the line $\theta = \frac{\pi}{6}$.

The region S, shown shaded in Figure 2, is bounded by the initial line, the line $\theta = \frac{\pi}{2}$, the line PR, the line QR and the curve C.

(b) Show that the area of the shaded region S can be given by

$$\frac{a^2}{8}\left(p\sqrt{3} + q\right)$$

where p and q are constants to be found. **(7)**

Question 8 continued

Question 8 continued

Question 8 continued

END OF PAPER **TOTAL 13 MARKS**

TOTAL FOR PAPER IS 75 MARKS