Surname	
Other Names	
Candidate Signature	

Centre Number						Candidate Number				

Examiner Comments		Total Marks

FURTHER MATHEMATICS

A LEVEL CORE PURE 1

CM

Bronze Set A (Edexcel Version)

Time allowed: 1 hour and 30 minutes

Instructions to candidates:

- In the boxes above, write your centre number, candidate number, your surname, other names and signature.
- Answer ALL of the questions.
- You must write your answer for each question in the spaces provided.
- You may use a calculator.

Information to candidates:

- Full marks may only be obtained for answers to ALL of the questions.
- The marks for individual questions and parts of the questions are shown in round brackets.
- There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to candidates:

- You should ensure your answers to parts of the question are clearly labelled.
- You should show sufficient working to make your workings clear to the Examiner.
- Answers without working may not gain full credit.

A2/FM/CP1

1 1 3 3 1 2 2 2 8 0 0 0 4

1 $$f(x) = x^3 - 8x^2 + 46x - 68$$

(a) Show that $f(2) = 0$. **(1)**

(b) Find the roots of the equation $f(x) = 0$. **(5)**

(c) Plot the roots of the equation $f(x) = 0$ on an Argand diagram. **(1)**

Question 1 continued

TOTAL 7 MARKS

2 The matrices **A**, **B** and **C** are defined such that

$$\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**. **(2)**

(b) Describe fully the sequence of transformations represented by the matrix **AC**. **(2)**

The matrix **M** is defined such that

$$\mathbf{M} = \begin{pmatrix} k & 3 \\ 1-k & k \end{pmatrix}$$

where k is a real constant.

The matrix **N = ABC – M**.

(c) Prove that the matrix **N** is never singular. **(4)**

Question 2 continued

TOTAL 8 MARKS

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3 (a) Show that $\dfrac{2}{r(r+1)}$ can be written in the form $\dfrac{A}{r}+\dfrac{B}{r+1}$, where A and B are constants to be found. **(2)**

(b) Hence, show that

$$\sum_{r=1}^{n}\frac{2}{r(r+1)}=\frac{pn}{qn+r}$$

where p, q and r are constants to be found. **(4)**

(c) Find the value n such that

$$\sum_{r=1}^{n}\left(12+\frac{6}{r\left(r+1\right)}\right)=65$$

(5)

Question 3 continued

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Question 3 continued

Question 3 continued

TOTAL 11 MARKS

4 The matrix **A** is defined such that

$$\mathbf{A} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

where a and b are constants.

(a) Use induction to prove that

$$\mathbf{A}^n = \begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$$

for every positive integer n. **(5)**

(b) Hence, find the values of the integers p and q such that

$$\begin{pmatrix} 4 & 0 \\ 0 & -3 \end{pmatrix}^p = \begin{pmatrix} 1024 & 0 \\ 0 & q \end{pmatrix}$$

(2)

Question 4 continued

TOTAL 7 MARKS

5 (a) Use de Moivre's theorem to prove that

$$\cos(nx) = \frac{e^{inx} + e^{-inx}}{2}$$

(2)

(b) Write down a similar expression for $\sin(nx)$. **(1)**

(c) Hence, show that

$$\cos(4x)\sin x = \frac{1}{2}\left(\sin(5x) - \sin(3x)\right)$$

(3)

(d) Find the exact value of

$$\int_0^{\pi} \left(9\pi\cos 4\theta \sin\theta + 11\pi \sin\theta\right)\, d\theta$$

(4)

Question 5 continued

Question 5 continued

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Question 5 continued

TOTAL 10 MARKS

6 The function f is defined such that

$$f(x) = k\sin[(4k - 1)x]$$

where k is an integer.

The mean value of $f'(x)$ on the interval $\left[0, \frac{\pi}{2}\right]$ is $\frac{4}{\pi}$.

(a) Find the value of k. **(4)**

(b) Hence, find the mean value of $f(x)$ on the interval $[0, \pi]$. **(3)**

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Question 6 continued

TOTAL 7 MARKS

7 The line l has the equation $\mathbf{r} = (2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$.

The plane Π_1 has the equation $4x + 2y - z = 5$.

(a) Find the coordinates of the intersection between the line l and the plane Π_1. **(3)**

(b) Calculate the size of the acute angle between the line l and the plane Π_1. **(3)**

The plane Π_2 contains the point $(\mathbf{j} + 2\mathbf{k})$, is parallel to the line l and is perpendicular to the plane Π_1.

(c) Find the Cartesian equation of the plane Π_2. **(5)**

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Question 7 continued

Question 7 continued

Do not write outside the box

Question 7 continued

TOTAL 11 MARKS

8

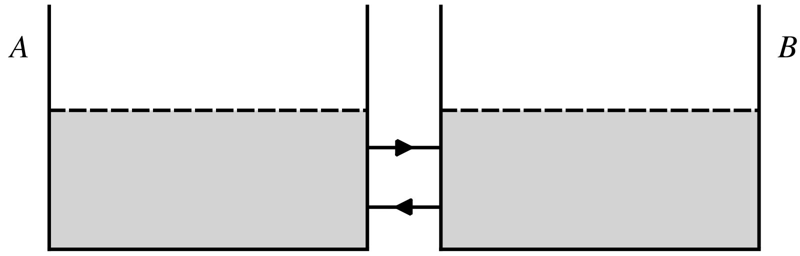

Figure 1

Figure 1 above shows two tanks, A and B, each of which holds 500 litres of water. A pipe pumps water from tank A to tank B at a constant rate of 5 litres per minute. At the same time, another pipe pumps water from tank B to tank A at the same constant rate of 5 litres per minute. At time $t = 0$, x_0 kg of a chemical X is dissolved into tank A and there is y_0 kg of the same chemical X in tank B.

The amount of chemical X in tank A, x, and the amount of chemical X in tank B, y, is modelled by the following system of differential equations

$$\frac{dx}{dt} = \frac{1}{100}(y - x)$$
$$\frac{dy}{dt} = \frac{1}{100}(x - y)$$

(a) (i) Show that $\frac{d^2y}{dt^2} + 0.02\frac{dy}{dt} = 0$. **(3)**

(ii) Find a general solution for the amount of chemical X in tank B. **(3)**

(iii) Hence, find a general solution for the amount of chemical X in tank A. **(2)**

(b) Using the initial condition, find, in terms of x_0 and y_0, an expression for

(i) the amount of chemical X in tank A

(ii) the amount of chemical X in tank B **(4)**

(c) Show that the limiting value for the amount of chemical in each tank is

$$\frac{1}{2}(x_0 + y_0)$$ **(1)**

(d) Explain, in the context of the problem, why the answer to part (c) is the expected value. **(1)**

Question 8 continued

Question 8 continued

Question 8 continued

END OF PAPER **TOTAL 13 MARKS**

TOTAL FOR PAPER IS 75 MARKS