A Level
Further Maths

Bronze Set A, Paper CP1 (Edexcel version)

Question	Solution	Partial Marks	Guidance
1 (a)	$f(2) = 2^3 - 8(2)^2 + 46(2) - 68$ $= 8 - 32 + 92 - 68$ $= -24 + 24$ $= 0$ (as required)	B1 **[1]**	Shows substitution of 2 into f and at least one intermediate step before arriving at 0
1 (b)	$f(x) = (x-2)(x^2 - 6x + 34)$ $x^2 - 6x + 34 = 0$ $\Rightarrow (x-3)^2 = -25$ $\Rightarrow x - 3 = \pm 5i$ $\Rightarrow x = 3 \pm 5i$ So roots are $3 \pm 5i$ and 2	M1 A1 M1* A1 A1 **[5]**	States or implies $(x \pm 2)$ is a factor of f and attempts to find other factor Correct other factor seen Complete method to solve their 3TQ for its complex roots M0 if their 3TQ has real roots $\pm 5i$ appearing in their complex roots All three roots stated
1 (c)		B1 **[1]**	All three roots plotted correctly on an Argand diagram Accept roots plotted as points or vectors Ignore any arguments or magnitudes
2 (a)	Enlargement of scale factor 3 about the origin	B1 B1 **[2]**	For “Enlargement” For “Scale factor 3” **AND “**centred/about at the origin/O”
2 (b)	Reflection in the x axis **followed by** an enlargement of scale factor 3 about the origin	B1 B1ft **[2]**	Reflection in x axis seen or associated with **C** Their “reflection” **followed by** their (a)

2 (c)	$\mathbf{ABC} = \begin{pmatrix} 3 & -6 \\ 3 & 9 \end{pmatrix}$	B1	Correct matrix for **ABC** seen or implied
	so $\mathbf{N} = \begin{pmatrix} 3-k & -9 \\ 2+k & 9-k \end{pmatrix}$		
	$\det \mathbf{N} = (3-k)(9-k)-(-9)(2+k)$ $= 27-12k+k^2+18+9k$ $= k^2-3k+45$	M1*	Finds **N** and uses a correct method to find its determinant
	Singular when det**N** = 0, i.e. $k^2-3k+45=0$	M1(dep*)	Sets their det**N** = 0 and uses a complete method to show it has no real solutions for k M0 if their 3TQ has real solutions
	But $(-3)^2-4(1)(45)=-171<0$, so the equation has no real solutions. So **N** is never singular	A1 **[4]**	Complete and convincing proof with no errors seen
3 (a)	$2 = A(r+1)+Br$		
	Let r = 0, then 2 = A, so A = 2 Let r = –1, then B = –2	M1	Complete method to find one of A or B
	So $\frac{2}{r(r+1)} = \frac{2}{r} - \frac{2}{r+1}$	A1 **[2]**	Correct values of A and B stated or shown embedded into expression. Final answer

3 (b)	$\sum_{r=1}^{n}\frac{2}{r(r+1)}$ $=\sum_{r=1}^{n}\frac{2}{r}-\frac{2}{r+1}$ $=\left(\frac{2}{1}-\frac{2}{2}\right)+\left(\frac{2}{2}-\frac{2}{3}\right)+\ldots+\left(\frac{2}{n-1}-\frac{2}{n}\right)+\left(\frac{2}{n}-\frac{2}{n+1}\right)$ $=2-\frac{2}{n+1}$ $=\frac{2n+2-2}{n+1}$ $=\frac{2n}{n+1}$	M1* A1 M1(dep*) A1 **[4]**	Writes out at least three terms in the series, including the first and the last and either the second or penultimate Telescopes correctly to obtain the sum of the series Uses a common denominator to obtain their sum of the series in the required form Correct answer or p, q and r stated
3 (c)	$\sum_{r=1}^{n}\left(12+\frac{6}{r(r+1)}\right)=65\Rightarrow\sum_{r=1}^{n}12+\sum_{r=1}^{n}\frac{6}{r(r+1)}=65$ $\Rightarrow 12n+\frac{6n}{n+1}=65$ $\Rightarrow 12n(n+1)+6n=65(n+1)$ $\Rightarrow 12n^2+12n+6n-65n-65=0$ $\Rightarrow 12n^2-47n-65=0$ $\Rightarrow (12n+13)(n-5)=0$ Since $n>0$, $n=5$	M1* A1 A1ft M1(dep*) A1 **[5]**	Attempts to use linearity (can be implied) Obtains term $12n$ Obtains correct sum of $\frac{6}{r(r+1)}$ ft their (b) Obtains a 3TQ and uses a complete method to solve it Correct value of n ISW If there are any additional solutions, do not ISW and give A0

4 (a)	Let $n = 1$, $\mathbf{A}^1 = \begin{pmatrix} a^1 & 0 \\ 0 & b^1 \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, so true for $n = 1$ Assume true for $n = k$, i.e. $\mathbf{A}^k = \begin{pmatrix} a^k & 0 \\ 0 & b^k \end{pmatrix}$ Then for $n = k + 1$, we have $\mathbf{A}^{k+1} = \mathbf{A}^k\mathbf{A}$ $= \begin{pmatrix} a^k & 0 \\ 0 & b^k \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ $= \begin{pmatrix} a^{k+1} & 0 \\ 0 & b^{k+1} \end{pmatrix}$ so true for $n = k + 1$ Therefore, <u>if true for $n = k$</u>, it has been shown to be <u>true for $n = k + 1$</u>. <u>Since true for $n = 1$</u>, it follows by induction that <u>it is true for all $n \in \mathbb{Z}^+$</u>	B1 M1* M1(dep*) A1 A1 **[5]**	Shows the statement is true for $n = 1$ and makes the statement Makes the assumption and shows intention to multiply $\mathbf{A}^k$ by $\mathbf{A}$ (either way around) Multiples matrices together with at least two correct entries Fully correct matrix multiplication Complete and convincing proof with no errors seen and a conclusion containing all underlined elements oe 'Since true for $n = 1$' in this conclusion can contribute to the 1st B1
4 (b)	$4^p = 1024 \Rightarrow p = 5$ Then $q = (-3)^5 = -243$	B1 B1ft **[2]**	Correct value of p Correct value of q ft their p
5 (a)	$\cos(nx) + i\sin(nx) = e^{inx}$ $\cos(nx) - i\sin(nx) = e^{-inx}$ Adding equations gives, $2\cos(nx) = e^{inx} + e^{-inx}$ $\Rightarrow \cos(nx) = \dfrac{e^{inx} + e^{-inx}}{2}$ **AG**	M1 A1 **[2]**	Writes down de-Moivre's Theorem and replaces x with $-x$ to obtain a second equation Adds the two equations and obtains the given result convincingly with no errors seen

5 (b)	$\sin(nx)=\dfrac{e^{inx}-e^{-inx}}{2i}$	B1 **[1]**	Correct expression oe
5 (c)	$\cos(4x)\sin x=\left(\dfrac{e^{4ix}+e^{-4ix}}{2}\right)\left(\dfrac{e^{ix}-e^{-ix}}{2i}\right)$ $=\dfrac{1}{2}\left(\dfrac{e^{5ix}-e^{3ix}+e^{-3ix}-e^{-5ix}}{2i}\right)$ $=\dfrac{1}{2}\left(\dfrac{e^{5ix}-e^{-5ix}}{2i}-\dfrac{e^{3ix}-e^{-3ix}}{2i}\right)$ $=\dfrac{1}{2}(\sin(5x)-\sin(3x))$	M1* M1(dep*) A1 **[3]**	Multiplies correct expression for $\cos(4x)$ with their (b) Obtains a 4 term expression with at least two terms correct Complete and convincing proof with no errors seen
5 (d)	$=\dfrac{9\pi}{2}\int_0^{\pi}(\sin(5\theta)-\sin(3\theta))d\theta+11\pi\int_0^{\pi}\sin\theta\, d\theta$ $=\dfrac{9\pi}{2}\left[-\dfrac{1}{5}\cos(5\theta)+\dfrac{1}{3}\cos(3\theta)\right]_0^{\pi}+11\pi[-\cos\theta]_0^{\pi}$ $=\dfrac{9\pi}{2}\left(\dfrac{1}{5}-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{3}\right)+11\pi(1--1)$ $=-\dfrac{6}{5}\pi+22\pi$ $=\dfrac{104}{5}\pi$	M1* A1 M1(dep*) A1 **[4]**	Uses part (b) to express integral in terms of $\sin(kx)$ terms only No need to use linearity – only looking for the correct expression Allow omission of factor of ½ from (b) for the M1 Correct integration of the correct expression Substitutes limits in the correct order (condone one sign slip) All $\cos(0)$ or $\cos(\pi)$ terms should be evaluated for this mark Correct exact value of the integral

6 (a)	$\frac{1}{\frac{\pi}{2}-0}\left[k\sin[(4k-1)x]\right]_0^{\frac{\pi}{2}}=\frac{4}{\pi}$ $k\sin\left[(4k-1)\frac{\pi}{2}\right]-k\sin(0)=2$ $\Rightarrow k(-1)=2$ $\Rightarrow k=-2$	M1 M1 B1 A1 **[4]**	Correct equation seen Limits do not need to be substituted for this mark M0 for an equation involving f' in integral form until the integration has been done and they obtain f again Substitutes limits into the equation in the correct order u Uses $\sin((4k-1)\pi/2)=-1$ Correct value of k
6 (b)	Hence mean value of f over the interval is $\frac{1}{\pi}\int_0^{\pi}-2\sin(-9x)dx$ $=-\frac{2}{\pi}\left[\frac{1}{9}\cos(-9x)\right]_0^{\pi}$ $=-\frac{2}{\pi}\left(-\frac{1}{9}-\frac{1}{9}\right)$ $=\frac{4}{9}\pi$	M1* A1ft A1 **[3]**	Expression for mean value of f in integral form (allow omission of $1/\pi$ and ignore any incorrect prefactors) ft their k Correct expression ft their k Correct final answer
7 (a)	$4(2+\lambda)+2(-3-2\lambda)-(1+3\lambda)=5$ $8+4\lambda-6-4\lambda-1-3\lambda=5$ $\Rightarrow\lambda=-\frac{4}{3}$ So coordinates of intersection are $\left(\frac{2}{3},-\frac{1}{3},-3\right)$	M1 A1 A1 **[3]**	Complete method to find λ Correct λ Correct coordinates of intersection

7 (b)	$\cos\theta = \frac{(\mathbf{i}-2\mathbf{j}+3\mathbf{k})\cdot(4\mathbf{i}+2\mathbf{j}-\mathbf{k})}{\sqrt{1^2+(-2)^2+3^2}\sqrt{4^2+2^2+(-1)^2}}$ $\Rightarrow \cos\theta = \frac{4-4-3}{\sqrt{14}\sqrt{21}} = -\frac{3}{\sqrt{14}\sqrt{21}}$ $\Rightarrow \theta = 100.076...$ so acute angle between line and the plane is 10.1°	M1 A1 A1 **[3]**	Correct unsimplified expression for the cosine of the angle between the line and the normal to the plane Correct angle between the line and the normal to the plane Correct acute angle
7 (c)	Need a vector perpendicular to $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ So components of this vector satisfy $x-2y+3z=0$ and $4x+2y-z=0$ i.e. $5x + 2z = 0$ Pick $x = 4$, then $z = -10$ and $y = -13$, so $4\mathbf{i} - 10\mathbf{j} - 13\mathbf{k}$ is perpendicular to the plane $4(0)-13(1)-10(2)=-33$, so the equation of the plane is $4x-13y-10z=-33$	M1 A1 A1 M1 A1 **[5]**	One of the two equations seen Solves correctly for the ratio between two of the variables Correctly finds a vector that is perpendicular to the plane Multiples of this should of course be accepted Method to use **their** perpendicular vector and the point the plane passes through to find "*d*" Correct equation of the plane oe **ALT for first 3 marks (use of cross product):** **M1 – complete method to find the cross-product between the two vectors** **A1 – any two components correct** **A1 – all three components correct**
8 (a) (i)	$x = y + 100\frac{dy}{dt} \Rightarrow \frac{dx}{dt} = \frac{dy}{dt} + 100\frac{d^2y}{dt^2}$ $100\frac{d^2y}{dt^2} + \frac{dy}{dt} = \frac{1}{100}y - \frac{1}{100}\left(y + 100\frac{dy}{dt}\right)$ $100\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = 0 \Rightarrow \frac{d^2y}{dt^2} + 0.02\frac{dy}{dt} = 0$ **AG**	M1* M1(dep*) A1 **[3]**	Attempts to differentiate the second equation with respect to *t* Substitutes in the first equation in **and** replaces the *x* by their "$y + 100y'$ " Complete and convincing proof with no errors seen

8 (a) (ii)	$m^2 + 0.02m = 0 \Rightarrow m(m + 0.02) = 0$, so $m = 0$ or $m = -0.02$ Hence, (since the equation is homogenous), the general solution is $y = A + Be^{-0.02t}$	M1 A1 A1 **[3]**	Forms the auxillary equation and uses a complete method to solve it Solution of the form $y = Ae^{\alpha t} + Be^{\beta t}$ Correct general solution
8 (a) (iii)	$x = A + Be^{-0.02t} + 100\left(-0.02Be^{-0.02t}\right)$ $\Rightarrow x = A - Be^{-0.02t}$	M1 A1 **[2]**	Attempts to use their y to find x Correct general solution for x
8 (b)	$x_0 = A - B$ $y_0 = A + B$ $2A = x_0 + y_0 \Rightarrow A = \frac{1}{2}(x_0 + y_0)$ and $-2B = x_0 - y_0 \Rightarrow B = \frac{1}{2}(y_0 - x_0)$ Hence:	M1 A1ft	**Parts (i) and (ii) of this question should be marked together** Forms two correct equations for their constants using the initial conditions Correct constants for their solutions ft their (a/ii) and (a/iii)
(i)	$x = \frac{1}{2}(x_0 + y_0) - \frac{1}{2}(y_0 - x_0)e^{-0.02t}$	A1	Correct particular solution for the amount of chemical X in tank A
(ii)	$y = \frac{1}{2}(x_0 + y_0) + \frac{1}{2}(y_0 - x_0)e^{-0.02t}$	A1 **[4]**	Correct particular solutions for the amount of chemical X in tank B
8 (c)	As $t \to \infty$, $e^{-0.02t} \to 0$, so $x \to \frac{1}{2}(x_0 + y_0)$ and $y \to \frac{1}{2}(x_0 + y_0)$ as $t \to \infty$ (as required)	B1 **[1]**	Convincing illustration with consideration of long-term behaviour of the exponential term
8 (d)	e.g. Both pipes between the tanks pump liquid at the same constant rate	B1 **[1]**	Explanation