
This worksheet is designed to give you extra practice on **algebraic proof**.

- 1 Write down algebraic expressions for
 - (a) an even number
 - (b) an odd number
 - (c) a multiple of four
 - (d) a positive number which leaves a remainder of 1 upon division by 5
 - (e) the sum of two consecutive even numbers
 - (f) the sum of two even numbers
 - (g) the product of two odd numbers
 - (e) the cube of a multiple of 6
 - (h) the sum of the squares of two consecutive multiples of 3
- 2 Prove algebraically that
 - (a) the sum of two consecutive numbers is odd
 - (b) the product of two even numbers is even
 - (c) the sum of any three consecutive numbers is a multiple of 3
- 3 (a) Prove algebraically that the sum of the squares of two consecutive odd numbers is **not** a multiple of 4.
(b) What is the remainder when the sum of the squares of two consecutive odd numbers is divided by 4? Is the number odd or even?
- 4 Johnny has an even number. He adds 3 to this number and then squares it.
The result is 81.
What even number did Johnny start with?
- 5 Prove algebraically that $(5n + 1)^2 - (5n - 1)^2$ is both a multiple of 4 and a multiple of 5 for positive integers n .
- 6 Anne has a number which is not a multiple of 3.
 - (a) Explain why it must be of the form $(3n + 1)$ or $(3n + 2)$, where n is an integer.
 - (b) Hence, by considering these two different cases, show that the square of Anne's number leaves a remainder of 1 when divided by 3.
- 7 (a) Prove that if n is an even integer, then $3n^2 + n + 14$ is even.
(b) Prove that if n is an odd integer, then $3n^2 + n + 14$ is even.
(c) What can you deduce about the number $3n^2 + n + 14$ for all integers n ?