1 Write down algebraic expressions for
  (a) an even number
  (b) an odd number
  (c) a multiple of four
  (d) a positive number which leaves a remainder of 1 upon division by 5
  (e) the sum of two consecutive even numbers
  (f) the sum of two even numbers
  (g) the product of two odd numbers
  (h) the sum of the squares of two consecutive multiples of 3

2 Prove algebraically that
  (a) the sum of two consecutive numbers is odd
  (b) the product of two even numbers is even
  (c) the sum of any three consecutive numbers is a multiple of 3

3 (a) Prove algebraically that the sum of the squares of two consecutive odd numbers is not a multiple of 4.
    (b) What is the remainder when the sum of the squares of two consecutive odd numbers is divided by 4? Is the number odd or even?

4 Johnny has an even number. He adds 3 to this number and then squares it. The result is 81.
   What even number did Johnny start with?

5 Prove algebraically that \((5n + 1)^2 - (5n - 1)^2\) is both a multiple of 4 and a multiple of 5 for positive integers \(n\).

6 Anne has a number which is not a multiple of 3.
   (a) Explain why it must be of the form \((3n + 1)\) or \((3n + 2)\), where \(n\) is an integer.
   (b) Hence, by considering these two different cases, show that the square of Anne’s number leaves a remainder of 1 when divided by 3.

7 (a) Prove that if \(n\) is an even integer, then \(3n^2 + n + 14\) is even.
    (b) Prove that if \(n\) is an odd integer, then \(3n^2 + n + 14\) is even.
    (c) What can you deduce about the number \(3n^2 + n + 14\) for all integers \(n\)?