GCSE: Iterations | CM

This worksheet is designed to give you extra practice on **using and applying iterative relations**.

1 The sequence a_n is defined such that $a_{n+1} = a_n + 3$ and $a_0 = 4$.

(a) Find the values of a_1, a_2 and a_3.

(b) What type of sequence is a_n?

2 The sequence u_n is defined such that $u_{n+1} = 2u_n - 3$ and $u_1 = -2$.

(a) Find the values of u_2, u_3 and u_4.

(b) Find the value of u_0.

3 Given that $x_{n+1} = 3 - \frac{x_n}{4}$ and $x_1 = 1$, find the values of

(a) x_2, x_3 and x_4,

(b) x_0.

Where appropriate, give each value to three decimal places.

4 Given that $x_{n+1} = x_n^{\,2} - \frac{7}{8}$ and $x_0 = 0$,

(a) write down the smallest term in the sequence.

(b) Find the values of x_1, x_2 and x_3, giving each value to four significant figures.

5 A deadly disease infects a population of plants.

The number of plants infected with the disease n days after the first infection, p_n, is given by

$$p_{n+1} = 4p_n + 6$$

and $p_0 = 3$.

(a) Interpret the meaning of p_0 in this context.

(b) Find the number of plants infected with the disease 2 days after the first infection.

There are 20000 plants in the population.

(c) Show that all the plants in the population will be infected with the disease after 6 days.

6 Markov invests £P into a bank account.

He is paid r% per annum compound interest and takes out £D a year from this account.

The amount of money in the account after n years is A_n.

(a) Show that

$$A_{n+1} = A_n\left(1 + \frac{r}{100}\right) - D$$

Markov invests £1000 into the account and the interest rate is 2%.

After 3 years, Markov has £143 in the savings account.

(b) How much money does Markov take out of the account each year? Give your answer to the nearest pound.

7 Here is a table of values for the curve with equation $y = f(x)$.

x	-3	-2	-1	0	1
$f(x)$	-10	-3	2	15	82

The equation $f(x) = 0$ has at least one solution in the interval $x = a$ and $x = b$.
Using the table, write down the value of a and b. Explain your reasoning.

8 Show that the equation $x^3 - 2x^2 - 7x + 1 = 0$ has a solution between $x = 3$ and $x = 4$.

9 Show that the equation $5x^3 + x^2 - 8x + 4 = 0$ has a solution between $x = -2$ and $x = -1$.

10 (a) Show that the equation $-3x^3 + 2x^2 + 4x + 5 = 0$ has a solution α between $x = 1.5$ and $x = 2$.

(b) Show that the equation $-3x^3 + 4x + 5 = 0$ can be re-arranged to give $x = \left(\frac{4x+5}{3}\right)^{\frac{1}{3}}$.

(c) Starting with $x_0 = 1.5$, use the iterative formula

$$x_{n+1} = \left(\frac{4x_n + 5}{3}\right)^{\frac{1}{3}}$$

three times to find an estimate for α.
Give your answer to three decimal places.

(d) The actual value of $\alpha = 1.5516$ to four decimal places.

(i) Find the percentage error in your estimate in (c).

(ii) Explain how you could make your approximation in (c) more accurate.

END OF TEST