Exercise 4Ai

Question 1:

a. 2
b. –11
c. $\frac{31}{2}$
d. $b-2$
e. – 1
f. 196
g. –8
h. 16

Question 2:

$$6 = 2^2 + a$$
$$\Rightarrow a = 6 - 4 = 2$$

Question 3:

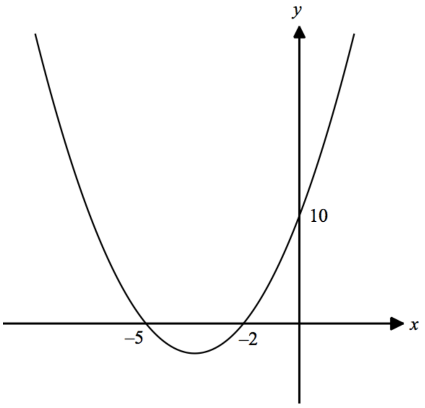
a. No, this would not be a function. A function takes an input and gives *an* output. If you have two output values for one input function, the machine/rule does not define a function.

b. Yes, this would be a function (think $y = x^2$).

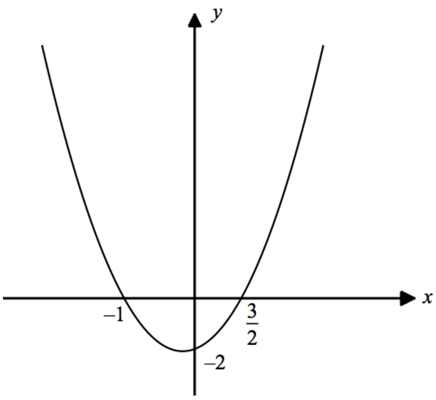
Exercise 4Aii

Question 1:

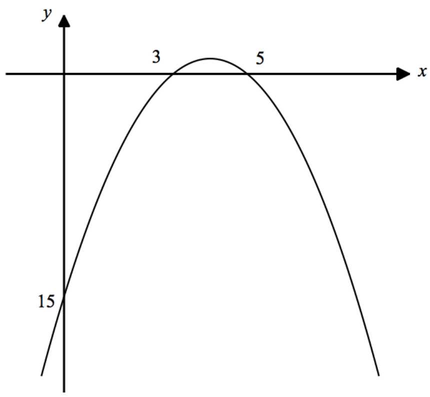
a.



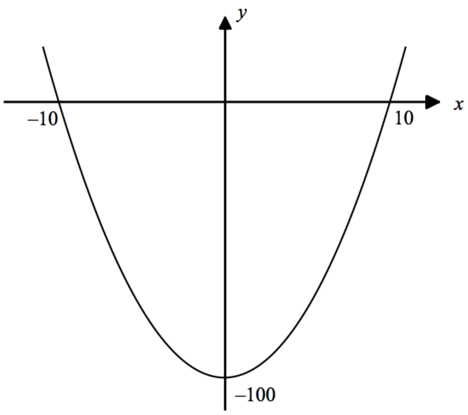
b.



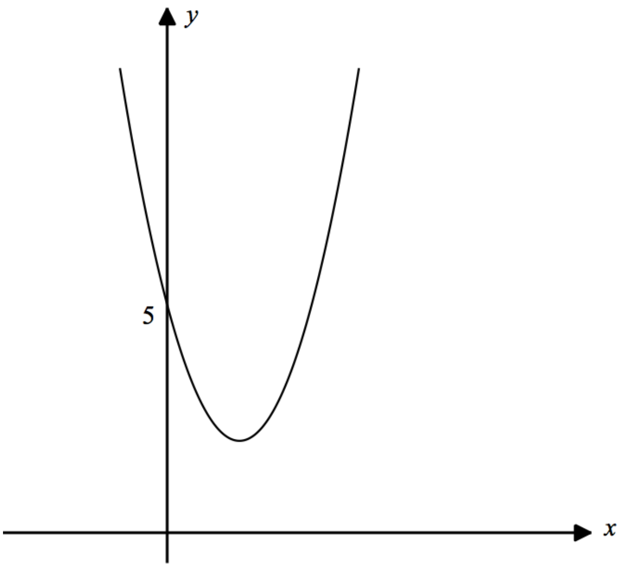
c.



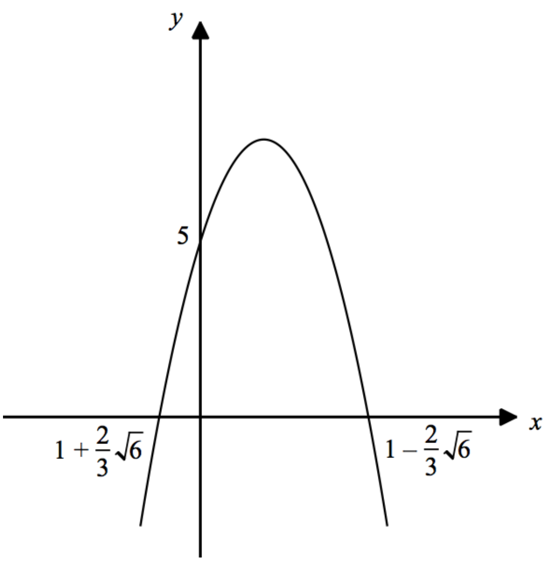
d.



e.



f.



Question 2:

a. (1, 3), minimum
b. (–2, –4), minimum
c. (9, 3), minimum
d. (7, 9), minimum
e. (1, 10), maximum
f.

$$\begin{aligned}4x^2-5x-8&=4\left(x^2-\frac{5}{4}x-2\right)\\&=4\left[\left(x-\frac{5}{8}\right)^2-\frac{25}{64}-2\right]\\&=4\left(x-\frac{5}{8}\right)^2-\frac{153}{16}\end{aligned}$$

so turning point, which is a minimum, has coordinates $\left(\frac{5}{8},-\frac{153}{16}\right)$.

g. $\left(-\frac{7}{4},\frac{113}{8}\right)$, maximum

h. $\left(\frac{1}{6},-\frac{59}{12}\right)$, maximum

Exercise 4Aii

Question 1:

a.

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-11)^2 - 4(3)(5) \\ &= 121 - 60 \\ &= 61\end{aligned}$$

61 > 0, so the equation has two (real) distinct roots.

(NB: in an exam, you would need to explicitly state that 61 > 0 **AND** that the equation has two **distinct** real roots. Two roots is **not** sufficient.)

Similarly, we find:

b. $\Delta = -8 < 0$, so no real roots

c. $\Delta = -24 < 0$, so no real roots

d. $\Delta = 97 > 0$, so two distinct real roots

e. Here we re-arrange to get in the standard form of a 3TQ.

$$\begin{aligned}&5x(x-3) = x^2 - 1 \\ &\Rightarrow 5x^2 - 15x = x^2 - 1 \\ &\Rightarrow 4x^2 - 15x + 1 = 0 \\ &\Rightarrow \Delta = (-15)^2 - 4(4)(1) = 209\end{aligned}$$

209 > 0, so there are two (real) distinct roots.

You will similarly find:

f. $\Delta = 0$, so there is one distinct real root OR two repeated real roots

g. $\Delta = -31 < 0$, so no real roots

h. $\Delta = 17 > 0$, so two distinct real roots. (There is a technicality here. We can't have $x = 2$, so when we re-arrange to get a quadratic and find it has two real roots, we have to be sure that 2 isn't one of them, which it isn't).

Question 2:

$$kx^2 + kx + (4-k) = 0$$

Equal roots $\Rightarrow \Delta = b^2 - 4ac = 0$

$$\Rightarrow k^2 - 4(k)(4-k) = 0$$

$$\Rightarrow k\{k - 4(4-k)\} = 0$$

$$\Rightarrow k(5k-16) = 0$$

$$\Rightarrow k = 0,\ k = \frac{16}{5}$$

{Accept if $k = 0$ omitted, as technically, the equation doesn't make much sense in that case, as we just get the contradiction $4 = 0$, but also condone if given.}

Question 3:

$$x^2 - 5x + 1 = kx + 2 \Rightarrow x^2 + (-5-k)x - 1 = 0$$

For no real roots, $\Delta = b^2 - 4ac < 0$

$$\Rightarrow (5+k)^2 - 4(1)(-1) < 0$$

$$\Rightarrow 25 + 10k + k^2 + 4 < 0$$

$$\Rightarrow k^2 + 10k + 29 < 0$$

Consider $k^2 + 10k + 29 = 0$

Then for this quadratic, the discriminant is –16. So it has no real roots. Since it is a positive U-shaped parabola, it is always above the y axis, so there are no values of k for which $k^2 + 10k + 29 < 0$.

Therefore, there are no values of k for which the equation has no real roots.

Extension q: can it have two equal roots? Or does it always have two distinct roots?

Question 4:

(a)

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-7)^2 - 4(1)(14) \\ &= -7\end{aligned}$$

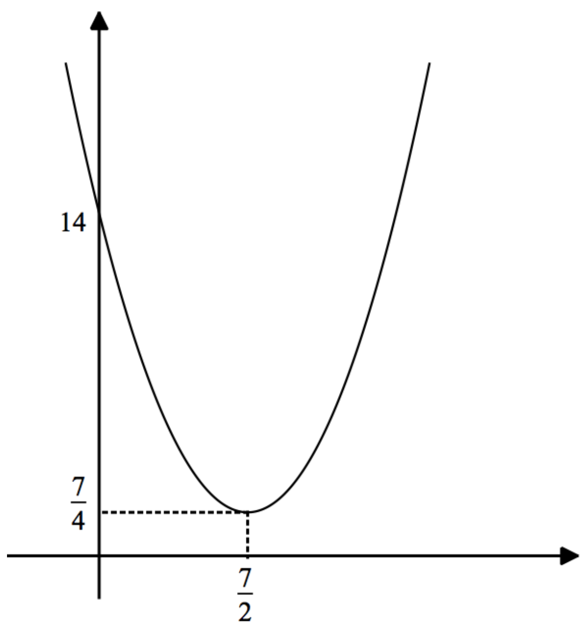
(b)

$$\begin{aligned}\mathrm{f}(x) &= x^2 - 7x + 14 \\ &= \left(x - \frac{7}{2}\right)^2 - \frac{49}{4} + 14 \\ &= \left(x - \frac{7}{2}\right)^2 + \frac{7}{4}\end{aligned}$$

(c)

For this sketch, we can use the previous parts to guide us. We know from part (a), the curve has no real roots so doesn't cross the x-axis. This reduces the shape of our curve to two options: U-shaped parabola with minimum point in 2nd quadrant (–ve x, +ve y) OR U-shaped parabola with minimum point in 1st quadrant (+ve x, +ve y).

Part (b) tells us which one of these it is, as we can see our curve has its minimum when x is positive and $= \frac{7}{2}$.



Question 5:

(a)

$$\begin{aligned} y &= (x+1)(x+6) \\ &= x^2 + x + 6x + 6 \\ &= x^2 + 7x + 6 \end{aligned}$$

so $a = 7, b = 6$

(b)

Crosses the y axis when $x = 0$, so in this case, crosses the y axis at $(0, 6)$

(c)

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= 7^2 - 4(1)(6) \\ &= 49 - 24 \\ &= 25 \end{aligned}$$

Sanity checks:

- you know the curve crosses the x axis twice (from the diagram!), so if you get a discriminant that is negative or = 0 here, you know that something has gone wrong
- you also know it factorises 'nicely', since it crosses the x axis at 'nice' points, so you expect the discriminant to be a perfect square.