GCSE: Completing the square

This worksheet is designed to give you extra practice on completing the square, using it to solve quadratic equations and finding minimum points on a quadratic curve.

1 Express each of the following expressions in the form $(x-a)^2 - b$, where a and b are constants to be found.

(a) $x^2 + 2x + 5$

(b) $x^2 - 4x + 1$ (c) $x^2 + 6x - 1$

(d) $x^2 - 12x + 8$

(e) $x^2 - 4x - 60$ (f) $x^2 + 12x + 32$ (g) $x^2 + 3x + 1$ (h) $x^2 + 8x - 2$

(i) $x^2 + 22x + 57$

(j) $x^2 + 10x$

(k) x(x-2)+1

(1) (x+2)(x-3)

2 Solve each of the quadratic equations below using completing the square.

(a) $x^2 + 5x + 6 = 0$

(b) $x^2 + 9x - 10 = 0$ (c) $x^2 - 8x + 12 = 0$ (d) $x^2 + 5x = 14$

(e) $x^2 + 6x - 59 = 0$

(f) $x^2 - 12x + 23 = 0$ (g) $2x + 3 = x^2$

(h) $x^2 - 10x + 26 = 8$

- 3 The quadratic curve C has the equation $y = (x a)^2 + b$.
 - (a) Explain why the curve has a minimum point when x = a.
 - (b) Write down the y coordinate of the minimum point on C.
 - (c) Find the coordinates of the point where C crosses the y axis.

The curve C meets the x axis provided $-b \ge 0$.

- (d) Explain the restriction $-b \ge 0$.
- 4 Find the real solutions to the quadratic equations below or prove that no real solutions exist.

(a) $x^2 + 8x - 10 = 0$ (b) $x^2 - 5x - 10 = 0$ (c) $x^2 - 5x + 20 = 0$ (d) $x^2 + 1 = 0$ (e) $x^2 + 11x - 2 = 0$ (f) $x^2 + 3x + 4 = 0$ (g) $x^2 - 11x - 60 = 0$ (h) $x^2 + 10x - 13 = 0$

5 The curve is defined such that y = f(x), where

$$f(x) = x^2 + ax + 5$$

and a is a constant.

- (a) Given that the curve passes through the point (1, 12), find the value of a.
- (b) Find the coordinates of the point where the curve crosses the y axis.
- (c) Find the coordinates of the point where the curve crosses the x axis.
- (d) (i) Find the coordinates of the turning point on the curve.
 - (ii) Is the turning point a minimum point or a maximum point? Justify your answer.

END OF WORKSHEET

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