This worksheet is designed to give you extra practice on completing the square, using it to solve quadratic equations and finding minimum points on a quadratic curve.

1 Express each of the following expressions in the form \((x-a)^2 - b\), where \(a\) and \(b\) are constants to be found.
   (a) \(x^2 + 2x + 5\)  (b) \(x^2 - 4x + 1\)  (c) \(x^2 + 6x - 1\)  (d) \(x^2 - 12x + 8\)
   (e) \(x^2 - 4x - 60\)  (f) \(x^2 + 12x + 32\)  (g) \(x^2 + 3x + 1\)  (h) \(x^2 + 8x - 2\)
   (i) \(x^2 + 22x + 57\)  (j) \(x^2 + 10x\)  (k) \(x(x-2)+1\)  (l) \((x+2)(x-3)\)

2 Solve each of the quadratic equations below using completing the square.
   (a) \(x^2 + 5x + 6 = 0\)  (b) \(x^2 + 9x - 10 = 0\)  (c) \(x^2 - 8x + 12 = 0\)  (d) \(x^2 + 5x = 14\)
   (e) \(x^2 + 6x - 59 = 0\)  (f) \(x^2 - 12x + 23 = 0\)  (g) \(2x + 3 = x^2\)  (h) \(x^2 - 10x + 26 = 8\)

3 The quadratic curve \(C\) has the equation \(y = (x-a)^2 + b\).
   (a) Explain why the curve has a minimum point when \(x = a\).
   (b) Write down the \(y\) coordinate of the minimum point on \(C\).
   (c) Find the coordinates of the point where \(C\) crosses the \(y\) axis.

   The curve \(C\) meets the \(x\) axis provided \(-b \geq 0\).
   (d) Explain the restriction \(-b \geq 0\).

4 Find the real solutions to the quadratic equations below or prove that no real solutions exist.
   (a) \(x^2 + 8x - 10 = 0\)  (b) \(x^2 - 5x - 10 = 0\)  (c) \(x^2 - 5x + 20 = 0\)  (d) \(x^2 + 1 = 0\)
   (e) \(x^2 + 11x - 2 = 0\)  (f) \(x^2 + 3x + 4 = 0\)  (g) \(x^2 - 11x - 60 = 0\)  (h) \(x^2 + 10x - 13 = 0\)

5 The curve is defined such that \(y = f(x)\), where
   \[f(x) = x^2 + ax + 5\]
   and \(a\) is a constant.
   (a) Given that the curve passes through the point \((1, 12)\), find the value of \(a\).
   (b) Find the coordinates of the point where the curve crosses the \(y\) axis.
   (c) Find the coordinates of the point where the curve crosses the \(x\) axis.
   (d) (i) Find the coordinates of the turning point on the curve.

      (ii) Is the turning point a minimum point or a maximum point? Justify your answer.

END OF WORKSHEET

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