## GCSE: Completing the square

This worksheet is designed to give you extra practice on completing the square, using it to solve quadratic equations and finding minimum points on a quadratic curve.

1 Express each of the following expressions in the form $(x-a)^{2}-b$, where $a$ and $b$ are constants to be found.
(a) $x^{2}+2 x+5$
(b) $x^{2}-4 x+1$
(c) $x^{2}+6 x-1$
(d) $x^{2}-12 x+8$
(e) $x^{2}-4 x-60$
(f) $x^{2}+12 x+32$
(g) $x^{2}+3 x+1$
(h) $x^{2}+8 x-2$
(i) $x^{2}+22 x+57$
(j) $x^{2}+10 x$
(k) $x(x-2)+1$
(l) $(x+2)(x-3)$

2 Solve each of the quadratic equations below using completing the square.
(a) $x^{2}+5 x+6=0$
(b) $x^{2}+9 x-10=0$
(c) $x^{2}-8 x+12=0$
(d) $x^{2}+5 x=14$
(e) $x^{2}+6 x-59=0$
(f) $x^{2}-12 x+23=0$
(g) $2 x+3=x^{2}$
(h) $x^{2}-10 x+26=8$

3 The quadratic curve $C$ has the equation $y=(x-a)^{2}+b$.
(a) Explain why the curve has a minimum point when $x=a$.
(b) Write down the $y$ coordinate of the minimum point on $C$.
(c) Find the coordinates of the point where $C$ crosses the $y$ axis.

The curve $C$ meets the $x$ axis provided $-b \geq 0$.
(d) Explain the restriction $-b \geq 0$.

4 Find the real solutions to the quadratic equations below or prove that no real solutions exist.
(a) $x^{2}+8 x-10=0$
(b) $x^{2}-5 x-10=0$
(c) $x^{2}-5 x+20=0$
(d) $x^{2}+1=0$
(e) $x^{2}+11 x-2=0$
(f) $x^{2}+3 x+4=0$
(g) $x^{2}-11 x-60=0$
(h) $x^{2}+10 x-13=0$

5 The curve is defined such that $y=\mathrm{f}(x)$, where

$$
\mathrm{f}(x)=x^{2}+a x+5
$$

and $a$ is a constant.
(a) Given that the curve passes through the point $(1,12)$, find the value of $a$.
(b) Find the coordinates of the point where the curve crosses the $y$ axis.
(c) Find the coordinates of the point where the curve crosses the $x$ axis.
(d) (i) Find the coordinates of the turning point on the curve.
(ii) Is the turning point a minimum point or a maximum point? Justify your answer.

