



Applied Mathematics

AS Level Mathematics Textbook

for the **Edexcel** Specification

First Edition

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ABOUT THIS BOOK

This book is designed to provide you with excellent preparation for your Edexcel AS Applied Mathematics examination:

- Written by subject experts
- Carefully designed for the Edexcel specification
- A focus on developing genuine understanding

How to use this textbook

Plenty of worked examples to show you how to apply key ideas. Callouts are used to explain key stages in more detail.

Break-it-down sections are used in more difficult questions to show you how to tackle and approach the harder questions.

SECTION 10.3

Worked Example 11

A light lift is attached to a vertical light inextensible string. The lift carries two masses *A* and *B*. The mass of *A* is 300 g and the mass of *B* is 500 g. *A* rests on top of *B* as shown in the diagram. The lift is raised vertically using the string at 2 m s^{-2} .

a. Find the tension in the string.
b. Find the force exerted on mass *A* by mass *B*.
c. Write down the force exerted on mass *B* by mass *A*.

a. Consider the whole system:
 $R(\uparrow): F = ma \Rightarrow 20 + T - T - R - R = (0.3 + 0.5)(2)$
 $\Rightarrow 20 - 2R = 1.6$
 $\Rightarrow 2R = 18.4$
 $\Rightarrow R = 9.2 \text{ N}$

b. Consider the forces on the box A
 $R(\uparrow): F = ma \Rightarrow R - 0.3g = (0.3)(2)$
 $\Rightarrow R = 0.3g + 0.6$
 $\Rightarrow R = 3.54 \text{ N}$

The force exerted on mass *A* by mass *B* is 3.5 N.

c. By Newton's Third Law, the force exerted on mass B by mass A is 3.5 N.

Tension and thrust - what's the difference?

Tension and thrust are two different forces and students can get confused between the two. Essentially, tension is a 'pulling' force and thrust is a 'pushing/compressing' force. For example, when rockets take off, they experience large amounts of upwards thrust. A string can be tense, but it cannot be in thrust, because it then would not be taut. A rod can be in either tension or thrust: if a rod is in tension, the two forces on it will be facing away from the centre of the rod. On the other hand, a rod in thrust is

FORCES AND NEWTON'S LAWS

essentially being 'squashed' and the forces will point towards the centre of the rod.

However, if you're not sure which direction the forces on the rod act, you can pick and the signs will tell you if your choice was right or wrong.

Worked Example 12

Two particles *P* and *Q* are connected by a light vertical rod to form a system, as shown in the diagram. The particle *P* has mass 0.5 kg and the particle *Q* has mass 0.8 kg. The rod is accelerated vertically upwards by a force of magnitude 30 N applied to *Q*. Find the thrust in the rod.

BREAK IT DOWN!

- All components of this system move in the same straight line, so we have the option to consider the whole system as a single particle.
- To find the thrust in the rod, we first need to find the acceleration of the system.

Consider the whole system:
 $R(\uparrow): F = ma \Rightarrow 30 - 0.5g - 0.8g = (0.5 + 0.8)a$
 $\Rightarrow a = 13.276 \dots$

Consider the particle P:
 $R(\uparrow): F = ma \Rightarrow T - 0.5g = 0.5a$
 $\Rightarrow T = 0.5g + 0.5(13.276 \dots)$
 $\Rightarrow T = 11.538 \dots$

So the thrust in the rod is 12 N.

Exercise 10C

1 A 5 kg lift is connected to a light inextensible string. The lift carries a box of mass 700 g, which rests on the floor of the lift. At time $t = 0 \text{ s}$, the lift is moving upwards at 8 m/s and starts to decelerate at a constant rate till coming to rest. Given that the tension in the string while the lift decelerates is 80 N, find the value of t when the lift comes to rest.

2 A car of mass 1500 kg is connected to a trailer of mass 800 kg using a tow-bar. The car produces a driving force of 4.4 kN and moves along a horizontal surface. The car experiences a constant resistance to motion of 600 N and the trailer experiences a constant resistance to motion of 450 N. By modelling the tow-bar as a light rod and the car and trailer as particles, find

a. the acceleration of the system b. the tension in the tow-bar

When the car is travelling at 23 m/s, the driving force is removed as the driver brakes for an emergency with maximum (constant) braking force. The system comes to rest in 2 seconds.

Each section has its own individual exercise so you can have plenty of practice with the key ideas before moving on.

Each chapter ends with a mixed exercise with plenty of questions to consolidate what you have learnt.

Chapters conclude with a summary of key points. These briefly outline the key ideas from the chapter to help you refresh your memory before moving on.

MIXED EXERCISE

- 3 The salaries of 9 employees in a company are listed in the table below along with the number of days of sick absence they have taken in the last year:

Salary (£ 000s)	25	19	24	17	28	16	25	35	72
Sick days	5	6	3	2	6	4	3	4	0

- Plot these data on a scatter graph.
- Describe the correlation between an employee's salary and the number of days of sick absence they take in this company.
- Comment on the assertion that lower-paid employees are more likely to take sick leave in this company.
- Explain why there is insufficient information to suggest that lower pay causes individual's to take more days of sick absence in this company.

7 Chelsea measures the arm spans of a group of 20 of the 30 children in her class. She summarises



REPRESENTATION OF DATA

- Explain whether the students were more accurate at guessing the number of sweets in the jar containing the same sweets or the jar containing different sweets.

Key points from Chapter 5

- It is often useful to represent data using a statistical diagram.
- Frequency polygons can be drawn by plotting the frequency of a class against the midpoint of the corresponding class.
- Cumulative frequency diagrams can be used to show running totals and estimate quartiles.
- In a histogram, the frequency of each class is proportional to the area of the corresponding bars on the histogram. Histograms are used to represent continuous data.

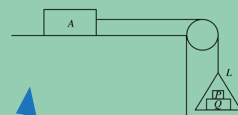
REFRESH AND REVIEW 2

This refresh and review is designed to help you consolidate, unify and practise the mechanics content on the course. If you are up for a challenge, try the challenge questions at the end.

- A 2 kg particle P is projected vertically upwards from the ground with speed 14 m/s. The particle moves freely under the influence of gravity before falling back to the ground and landing in a pool of water. As the particle enters the water, it is subject to a constant resistance to motion of 30 N.
 - Find the maximum height P reaches above the ground.
 - Find the velocity of P at the instant it enters the water.
 - Calculate the distance travelled by P in the water.
- A rollercoaster starts from rest and accelerates at a m s⁻² until it reaches a speed of 60 m s⁻¹. The rollercoaster then travels at this speed for 10 m before decelerating to rest in 3 seconds. The rollercoaster track is 250 m, find
 - the time the rollercoaster ride lasts
 - the value of a
- Three forces $\mathbf{F}_1 = (1 + j)$ N, $\mathbf{F}_2 = (3i + 5j)$ N and $\mathbf{F}_3 = (ki + j)$ N act on a particle P , where k is a constant. The resultant force on the particle is parallel to the resultant force of \mathbf{F}_1 and \mathbf{F}_2 .
 - Find the value of k .
 - Find the magnitude of the resultant force acting on P .
 - Find the direction of the resultant force acting on P , giving your answer as an angle measured clockwise from the unit vector \mathbf{j} .
- A particle moves in a straight line. The velocity of the particle, v m s⁻¹, at time t s is given by $v = 8\sqrt{t} - t^2$ for non-negative t .
 - Find the maximum velocity of the particle.
 - Find the time when the particle is at rest.
- Two masses, A and B , are connected by a light inextensible string that passes over a fixed smooth pulley. The mass of A is 4m kg and the mass of B is 6m kg. The masses hang freely with A at a height of 3 m above the ground and B at a height of h m above the ground. The system is released from rest.
 - Find the acceleration of each particle and the tension in the string.
 - Given that the particle B hits the ground at 2 m/s, find the value of h .
 - Find the maximum height the particle A reaches above the ground.
 - In the modelling setup, the string was described as being light and inextensible.
 - Explain what you understand about the string being light.
 - State how you have used the fact that the string is inextensible in your calculations.
- A 3 kg particle moves on the x axis. The velocity, v m s⁻¹, of the particle at time t s is given by $v = 4t^2 - 8t + 11$, $t \geq 0$.
 - Find the magnitude of the resultant force acting on the particle when $t = 4$.
 - Show that the particle only moves in the one direction for all t .
- Particles P and Q , of mass 2 kg and 5 kg respectively, are connected by a light inextensible string. The particle P lies on a rough horizontal surface x m away from a small smooth pulley, which is fixed to the edge of the surface. The string passes over the pulley and the particle Q hangs freely, 0.85 m above the ground. A frictional force of F N opposes the motion of P . The system is released from rest and the tension in the string is 4g N.
 - Find the acceleration of each particle and the tension in the string.
 - Find the time when the particle Q hits the ground.
- A particle P moves along a straight line. The speed of the particle at time t s is v m s⁻¹, where $v = at^2 + bt + c$, $t \geq 0$ and a , b and c are constants. When $t = 2$, the speed of P attains its minimum value. When $t = 0$, $v = 10$ and when $t = 2$, $v = 4$. Find
 - the acceleration of P when $t = 5$
 - the distance travelled by P during the third second of its motion.
- A lift rises vertically from rest with constant acceleration. After 5 seconds, the lift is moving upwards with a velocity of 2 m/s. It then moves with this constant velocity for 5 seconds. The lift then slows down uniformly, coming to rest after it has been moving for a total of 14 seconds.
 - Sketch a velocity-time graph for the motion of the lift.
 - Find the total distance travelled by the lift. The lift is raised by a single vertical cable. The mass of the lift is 450 kg.
 - Find the maximum tension in the cable.
 - Explain why your answer is a maximum.
- A 4 kg particle is projected vertically upwards at 18 m/s. As the particle moves through the air, it is subject to a constant resistance to motion of D N. The particle reaches its maximum height above the ground 1.5 seconds after projection.
 - Find the value of D .
 - Find the maximum height of the particle above the ground. James says that, "the total time the particle is in the air is $1.5 \times 2 = 3$ seconds."
 - Explain, with reference to the forces acting on the particle, why James is wrong.
 - State, with reason, whether the time taken by the particle to fall to the ground from its maximum height is greater than or equal to 1.5 s.
 - Find the time the particle is in the air.

Stretch & Challenge

- Alice and Ben are both standing in a lift. The mass of the lift is 300 kg. The lift accelerates upwards at a constant rate. The normal reaction forces exerted by the lift on Alice and Ben have magnitude 650 N and 460 N respectively. The tension in the cable pulling the lift upwards is 4000 N. Find the mass of Alice and the mass of Ben.
- A particle travels in a straight line. The acceleration of the particle at time t s is a m s⁻², where $a = 4t - 3$. Given that the particle travels 40 m in the third second of its motion, find its velocity when $t = 5$.
- A 4 kg particle A is connected to a light lift L by a light inextensible string. The string passes over a smooth pulley that is fixed to the edge of a rough horizontal table. The particle A lies on the table and the lift L hangs freely, as shown in the diagram. The frictional force between A and the table has constant magnitude of 6 N. The lift carries two blocks, P and Q , with P on top of Q . The mass of block P is 3 kg and the mass of block Q is 8 kg.



The system is released from rest. By modelling blocks as particles and ignoring the effects of air resistance, find

- the acceleration of the lift
- the tension in the string
- the force exerted by P on Q
- the magnitude and direction of the force exerted on the pulley by the string.

Refresh and review exercises half-way through and at the end of the book to test many topics at once.

Each refresh and review exercise ends with a selection of stretch and challenge questions to help develop your understanding even further.

Learning goals for Chapter 2


By the end of this chapter, you should be able to:

- understand the importance of sampling
- describe the different methods of sampling
- evaluate different sampling methods in various contexts
- discuss the limitations of sampling

Prior Knowledge

None! We will be introducing the idea of sampling to you and will assume no background knowledge on sampling. However, we will refer to the large data set, so you should make sure you understand the variables involved in it.

Chapter 2: Sampling



In this chapter, we will be exploring the methods statisticians may use to collect data. The collection of data is very important in many industries, but the method of approach needs to be thought through carefully to meet the statistician's requirements.

2.1 The statistician's goal

In the real world, statisticians work to design and create surveys and experiments. From these experiments, they collect, process and analyse data and then interpret the results of these experiments and surveys. For example, in a pharmaceutical company, statisticians might develop and evaluate the results of clinical trials to determine the safety and effectiveness of new medicines. Environmental statisticians might use data to assess the changes in climate patterns and make predictions about the future.

As we can see, statisticians work in a variety of different contexts and each context may require its own unique approach. In statistics, a **population** is the whole set of items that are of interest. If we want to collect information about a population, we can take a census or a sample. The information obtained is known as **raw data**.

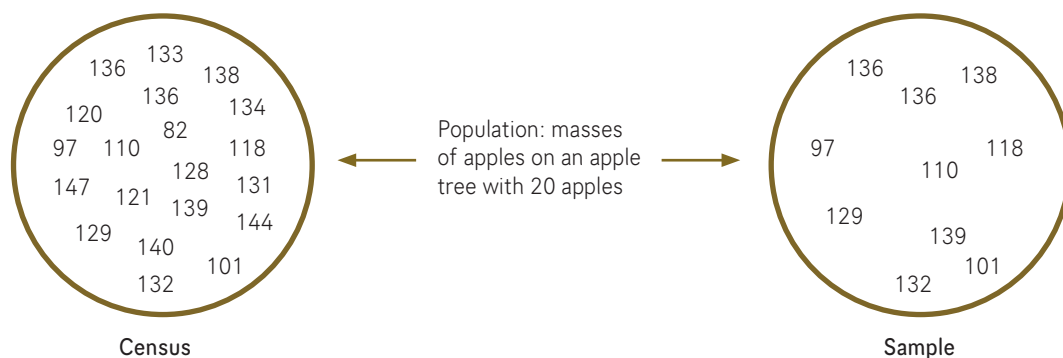
If you were investigating the radii of apples on an apple tree, then your population would consist of the radii of all of the apples on the apple tree.

A census

A **census** observes or measures information about every member of a population. For example, if we wanted to measure the mean height of students in a year group, we could measure the height of each individual student. A popular, well-known census is the one conducted by the British Government every 10 years. This census involves known householders receiving a census form that they must complete and return by a certain date. The form records information such as age, occupation and so on.

A sample

A **sample** observes or measures information from a subset of the population. This data is then used to find out information about the population as a whole. This is known as a **sample survey**. For example, suppose we wanted to measure the mean height of students in a year group. Instead of taking a census, we could measure the heights of some of the students in the year group and use that data to estimate the mean height. This is useful as it saves time. Although a year group of 200 students is not necessarily too large a population, if we wanted to find the average height of a 17 year old male, our population size would now be much larger. Taking a census here would be very time consuming.



A sample that is chosen well will be truly representative of a population. To choose a sample well, it should be free from bias. Each sampling unit must be selected at random. In other words, don't walk into a classroom and pick the 10 tallest students as your sample; perhaps use a number generator to select the n th student on a sampling frame, such as a register.

The sample size we use does not necessarily depend on the population size. You do not necessarily need to use a larger sample size if the population is larger. However, the sample size you choose does depend

As a result, different samples can result in different conclusions being made about a population. It is important that researchers bear this in mind when analysing the data collected from their samples.

Mixed Exercise

- 1 In a town, there has been an increased number of individuals drink-driving. The local police station decides to stop and test more drivers for alcohol in a new attempt to reduce the number of drink-driving cases.
 - a. Suggest, in context, why it is **not** a good idea for the police officers to stop and test all of the drivers they encounter for alcohol.
 - b. Explain why opportunity sampling is a good approach that the police officers can take.
 - c. Suggest why a simple random sample is not completely feasible in this context.
- 2 A company wants to improve the current levels of fitness of its employees. As a first step, the company organises to administer a health and fitness test for a sample of all of its employees to assess their current levels of fitness. The company has a **total** of 7000 employees.
 - a. Describe how random numbers could be used to select a systematic sample of 28 employees.
 - b. Give a reason why this sampling method might not be representative of all of the company's employees.
- 3 Emma is a biologist. She wants to study the numbers of different plant species in a field. She plans an investigation to do this using a square grid (quadrat). She will place her quadrat in a random position on the field, move the quadrat by 2 m and record the species touching the corners of the quadrat. She uses until she has sampled across the entire field.
 - a. State the type of sampling method Emma is using.

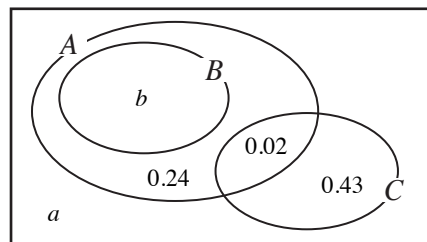
Emma completes her investigation. The data that she collects does not show the presence of a plant species that is scattered on the field irregularly.

 - b. Suggest how Emma could modify her procedure to make her sample more representative of the entire field.
- 4 Sarah records the heights of students at her school as part of a research project. She wants to compare the heights of students in Years 9, 10 and 11 and estimate the mean growth a student makes between these years. She also wants to stratify her investigation by gender. Sarah measures the heights of all of the males in the Year 9, Year 10 and Year 11 school basketball team.
 - a. Comment on Sarah's method.
 - b. Describe fully an improved random method Sarah could use to obtain data she can analyse to estimate mean growth a student makes between Years 9, 10 and 11.
- 5
 - a. Explain what you understand by a population.
 - b. State **one** disadvantage of using a census to survey a population as opposed to a sample. The managing director of a company wants to survey what its workers think about the factory's canteen. The company has 3000 employees. 70% of the employees eat meat and 20% of the employees are vegetarians. 10% have other dietary requirements.
 - c. Explain briefly how the director could select a sample of 60 workers using
 - i. simple random sampling ii. stratified sampling iii. opportunity sampling
 - d. State **one** reason the manager may want to use quota sampling as opposed to stratified sampling.

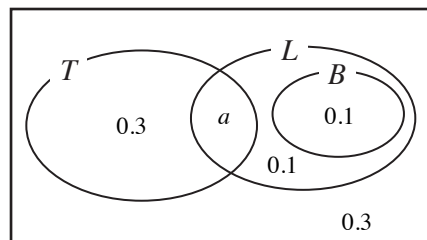
Key points from Chapter 2

- 1 In statistics, a **population** is the whole set of items that are of interest.
- 2 A **census** surveys every element of the population.
- 3 A **sample** surveys a subset of the population.
- 4 A census is fully accurate but takes more time and resources than a sample. It also results in more data, which can be more difficult to process. It also cannot be used where testing leads to the destruction of the product - for example, testing the lifetime of a bulb.
- 5 A sample takes less time to complete than a census and less people need to respond. However, it is less accurate and can miss niche characteristics of a population.
- 6 A sample of size n is called a **simple random sample** if every other sample of size n has an equal chance of being selected. This is sampling without replacement.
- 7 A simple random sample can be formed using random numbers together with a sampling frame.
- 8 Systematic sampling and stratified sampling are other types of random sampling techniques.
- 9 Quota and opportunity (convenience) sampling are examples of non-random sampling techniques. They are useful because they are quicker and do not require a sampling frame.
- 10 When sampling, it is important to be aware that your data is not fully accurate. Different samples can lead to different conclusions about a population.

- 3** The Venn diagram shows the probabilities for three events A , B and C . Given that the probability of both A and B occurring is 0.08,
- find the values of the constants a and b .
 - Explain why the events A and B are not statistically independent.
 - Describe the relationship between the events B and C .



- 4** The following Venn diagram shows the probabilities of the event L , “Charlie is late to work”, the event T , “Charlie is stopped by at least one red traffic light on his way to work”, and the event B , “Charlie’s boss witnesses that he is late”.
- Explain why B is contained entirely within L .
 - Find the value of a , the probability that T and L occur together.
 - Determine whether the events T and L are statistically independent. Justify your answer.



- 5** At a school, the probability that a randomly selected Y12 student studies Maths is 0.5, Geography is 0.3 and the Extended Project is 0.4. No students who study Maths or Geography also study the Extended Project. 7% of all Y12 students at the school do not study Maths, Geography or the Extended Project. By forming and solving simultaneous equations, or otherwise, find the probability that a student studies both Maths and Geography.

6.4 Discrete and continuous distributions

If the outcome of an experiment such as the value of the roll of a dice is mapped to a variable, say X , then the possible values of X and their associated probabilities form a probability distribution. We often say that X is a **random variable**. Where the variable takes a small number of discrete values, it is often convenient to tabulate the outcomes. There are certain discrete probability distributions that behave in a predictable manner - we will see a common one of these in the next section: the Binomial Distribution.

For a random variable X ,

- we use a lower case x to represent a particular value of X
- we use the notation $P(X = x)$ to represent the probability that X equals a particular value x

Random variables can either be discrete or continuous. Discrete random variables can only take values on a discrete scale. Continuous random variables are those that can take any value on a continuous scale.

Worked Example 6

Give an example of

- a discrete random variable
- a continuous random variable

- the number of heads obtained from tossing a coin 6 times
- the height of a group of boys

To specify a discrete random variable completely, you need to know the set of values it can take and the associated probabilities.

With this information, you can draw a table to show the probability of each value the random variable can take. This table is known as a **probability distribution**.

You can also specify a random variable using a function. For discrete random variables, the function is known as the **probability mass function**. For continuous random variables, the function is known as the **probability density function**.

Worked Example 7

The discrete random variable X has the following probability distribution:

x	0	1	2	3
$P(X = x)$	0.1	0.24	k	0.14

where k is a constant.

- Find the value of k .
- Find $P(1.5 < X < 5)$.

$$\text{a. } 0.1 + 0.24 + k + 0.14 = 1$$

$$\Rightarrow k = 1 - 0.48 = 0.52$$

$$\begin{aligned} \text{b. } P(1.5 < X < 5) &= P(2 \leq X \leq 3) \\ &= P(X = 2) + P(X = 3) \\ &= 0.52 + 0.14 \\ &= 0.66 \end{aligned}$$

Form and solve an equation in k using that the sum of all probabilities is 1.

X is a discrete random variable that takes the values 0, 1, 2 and 3. In the range $(1.5, 5)$, X can take the values 2 and 3, so we add together those probabilities.

Worked Example 8

The discrete random variable X has the following probability function:

$$P(X = x) = \begin{cases} 0.5(x-1) & x = 1, 2 \\ k & x = 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

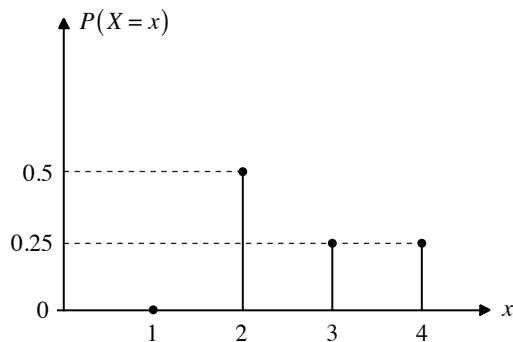
Since X is constant for $x = 3$ and 4, it is said to have the **discrete uniform distribution** for $x = 3$ and 4.

where k is a constant. Find the value of k .

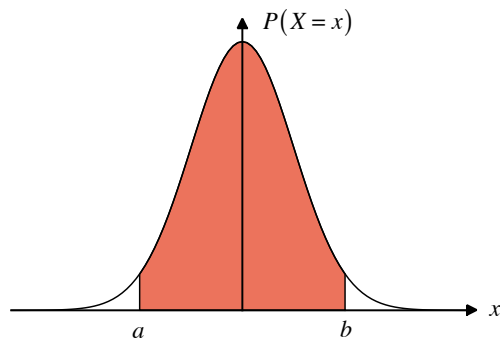
$$\begin{aligned} 0.5(1-1) + 0.5(2-1) + k + k &= 1 \\ \Rightarrow 2k &= 1 - 0.5 \\ \Rightarrow k &= 0.25 \end{aligned}$$

Form and solve an equation in k using that the sum of all probabilities is 1.

We can plot the probability function of a random variable. For example, for the discrete random variable in Worked Example 7, the probability function is shown in the diagram on the next page. Since the function is only defined at discrete values, the function is not continuous and it is not particularly useful other than to see the shape of the distribution.



Probability mass function for Worked
Example 7



Probability density function for a
continuous random variable

For a continuous random variable, the function will be continuous (often piecewise continuous). In particular, the area under the curve between two limits $x = a$ and $x = b$ represents the probability of the random variable taking a value between the limits. This is one important use of integration in probability. You will not be expected to use integration or to calculate the area under the probability density function, but you should understand what the area represents.

Exercise 6D

- 1 State whether the following variables are random variables. If it is a random variable, state further if it is discrete or continuous.

- time taken for 100 individuals to run 100m
- number of people that like the colour green from a sample of 30 people
- number of months in a year
- amount of rainfall, in mm, in Heathrow on a particular day in May 2015
- daily mean total cloud, in oktas, in Hurn on a particular day in 1987

- 2 The discrete random variable Y has probability distribution:

y	-3	-1	2	3
$P(Y = y)$	k	$\frac{8k-1}{4}$	$2k$	0.1

where k is a constant.

- Find the exact value of k .
- Determine whether Y is more likely to be positive or negative.
- Write down the value of $P(-5 \leq Y < -3)$.

- 3 The discrete random variable Y has probability function:

$$P(Y = y) = \begin{cases} \frac{ky}{4} & y = -2, -1, 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

- Find the probability distribution of Y .
Given that $X = 2Y - 1$,
- find $P(-2 \leq X < 2)$.

In Worked Example 2, our significance level was 10%. However, the probability of obtaining a value 28 or more is actually only 7.8%. This is often called the **actual significance level** of the hypothesis test or the **p-value**. It is also the probability of rejecting the null hypothesis given that it is true.

Worked Example 4

In Karan's restaurant, 1 in 4 customers order a chicken curry. Karan modifies the recipe of this dish. Several weeks later, he takes a sample of 30 of his customers' orders to see if the number of customers ordering chicken curry has changed.

- Using a 5% level of significance, find the critical region for this test. The probability in each tail should be as close to 2.5% as possible.
- State the actual significance level of the test.

!BREAK IT DOWN!

- This is a two-tailed test. So our critical region will consist of two parts.
- When doing a two-tailed test, you use half of the significance level for each tail, as the question has prompted.

a. Let X be the number of customers that buy a chicken curry and p the probability of a customer buying a chicken curry.

$$H_0 : p = 0.25, \quad H_1 : p \neq 0.25$$

b. Assume H_0 to be true, then $X \sim B(30, 0.25)$

Consider the lower tail:

$$P(X \leq 2) = 0.0106$$

$$P(X \leq 3) = 0.0374$$

0.0374 is closest to 0.025, so 3 is the critical value for the lower tail

Consider the upper tail:

$$P(X \geq 12) = 0.0507$$

$$P(X \geq 13) = 0.0216$$

0.0216 is closest to 0.025, so 13 is the critical value for the upper tail

\therefore critical region : $x \leq 3$ or $x \geq 13$

b. Actual significance level is

$$0.0216 + 0.0374 = 0.059 \text{ or } 5.9\%.$$

Define X and p and write your hypotheses in the correct form.

Get these values from tables or your calculator.

Read the question carefully: even though 0.0374 is above 0.025, it is still the closest value.

State clearly what the critical region is.

Exercise 7B

- Each day, a company receives about 40 emails. Of those 40 emails, it is expected that 18 are orders. A manager believes that the company are receiving less orders than usual. On a random day, she records the number of email orders the company receives out of the 40 emails. Using a 5% level of significance, find the critical region to test the manager's hypothesis. State your hypotheses clearly.

Learning goals for Chapter 10

By the end of this chapter, you should be able to:


- state and apply Newton's laws of motion
- solve problems involving forces
- solve problems involving connected particles

Prior Knowledge

You do not need to have any prior knowledge of forces in mechanics for this chapter. However, you need to be aware of the modelling assumptions in mechanics. You also need to be able to confident with the vectors content from AS Pure.

Chapter 10:

Forces and Newton's Laws

A large construction crane is silhouetted against a bright blue sky with scattered white clouds. The crane's lattice structure and cables are clearly visible. In the lower right, the sun is low on the horizon, creating a warm orange glow that illuminates the bottom of the crane's structure.

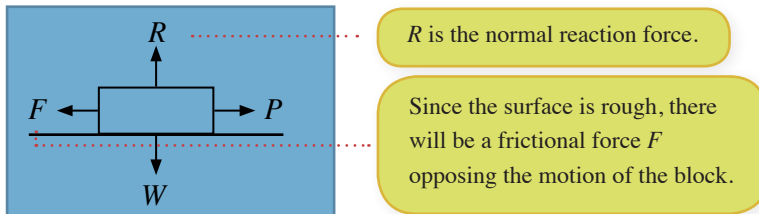
Newton's laws of motion are fundamental for the mechanics we will be dealing with. We will be looking at what these laws are and how we can use them to solve problems in mechanics. By the end of this chapter, you will understand why a ball projected in the air will eventually come down and much more.

10.1 Newton's Laws

When solving problems with forces, it is important to be able to draw force diagrams.

Worked Example 1

A block of weight W is being pulled to the right by a force P across a rough horizontal surface. Draw a force diagram that shows all of the forces acting on the block.



Newton's First Law of Motion

Newton's 1st Law (N1) states that every body will continue in its state of rest or uniform motion unless acted upon by an external force.

Newton's Second Law of Motion

Before we state Newton's 2nd Law (N2), we need to define some terms precisely:

- the **mass** of an object is its ability to resist changes in velocity
- the **resultant force** on an object is the vector sum of all of the forces acting on it
- an object is said to be in **equilibrium** if the resultant force on the object is 0. An object in equilibrium does not accelerate.

Newton's second law states that the resultant force acting on a particle is equal to the mass of the particle multiplied by the acceleration of the particle in the direction of the resultant force.

Mathematically, we can write $F = ma$.

Newton's Third Law of Motion

Newton's 3rd Law (N3) is that "to every action, there is an equal and opposite reaction". This law is required by N1 and N2; consider an object at rest and imagine it to have two halves, which exert forces on each other. Unless these forces are equal and opposite, the resultant force would be non-zero, causing the object to accelerate by N2 which violates N1. Hence, N3 is required by N1 and N2.

Worked Example 2

The force diagram shows the forces acting on a 3 kg particle P .

Find **a.** the resultant force acting on P

b. the acceleration of P .

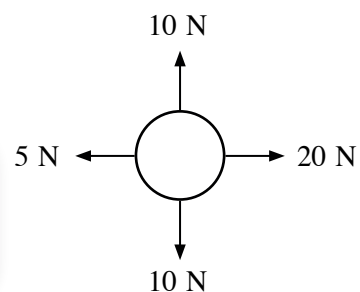
a. $R(\uparrow): 10 - 10 = 0 \text{ N}$

$R(\rightarrow): 20 - 5 = 15 \text{ N}$

So the resultant force acting on P is 15 N to the right.

Consider the forces acting vertically: + 10 N acts upwards and - 10 N acts downwards, so the net vertical force is 0 N.

Notation: we write $R(\uparrow)$ to indicate that we are considering vertical forces with upwards positive.



b. Using N2,

$$F = ma \Rightarrow 15 = 3(a)$$

$$\Rightarrow a = \frac{15}{3}$$

$$\Rightarrow a = 5 \text{ m s}^{-2}$$

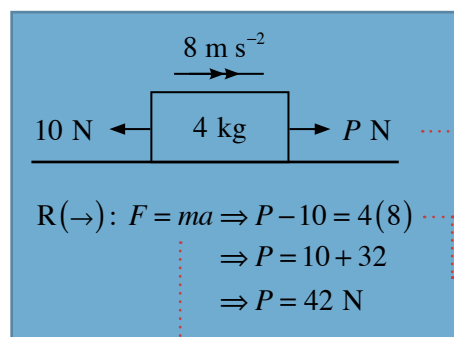
So P accelerates at 5 m s^{-2} to the right.

Use Newton's 2nd Law to find the acceleration of the particle.

The particle will accelerate in the direction of the resultant force.

Worked Example 3

A book of mass 4 kg accelerates at 8 m s^{-2} along a rough horizontal table by a force $P \text{ N}$. Given that the magnitude of the force due to the friction is 10 N , find P .



Draw a diagram. Remember that P will act in the direction of acceleration and friction will act in the opposite direction to motion.

Resolve the forces horizontally. It is usually easier to take the direction of motion to be positive.

The resultant vertical force is $P - 10$ to the right. If we took left to be positive, our equation would be $10 - P = 4(-8)$.

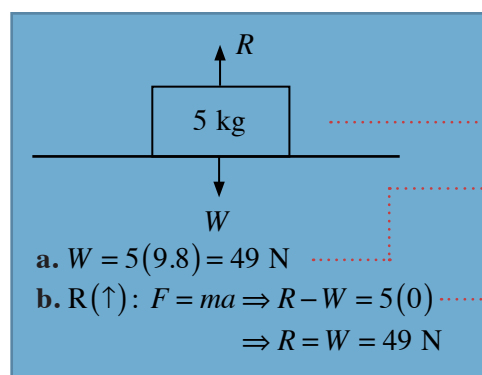
The gravitational force of attraction causes objects to be attracted to each other. The force acting on an object due to gravity is called the **weight** of the object. Note that weight and mass are not the same physically - even though the terms are used interchangeably in daily life. Using N2, we can formulate that the weight of an object, $W \text{ kg}$, is related to the mass of the object, $m \text{ kg}$, by $W = mg$, where g is the acceleration due to gravity; g depends on location, but unless told otherwise, assume $g = 9.8 \text{ m s}^{-2}$.

Worked Example 4

A body of mass 5 kg lies on a smooth surface. Find

a. the weight of the body

b. the normal reaction force exerted by the surface on the body



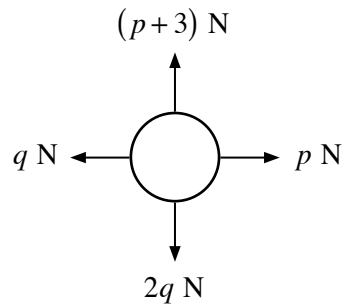
Draw a diagram.

Use $W = mg$.

Resolve vertically. Since the object is at rest vertically, its vertical acceleration is 0.

Worked Example 5

The force diagram shows the forces acting on a particle P . Given that P is in equilibrium, find the values of p and q .

**!BREAK IT DOWN!**

- Since P is in equilibrium, the resultant force is 0.
- Hence, all vertical and horizontal forces must balance.

$$R(\uparrow): F = ma \Rightarrow p + 3 = 2q$$

$$R(\rightarrow): F = ma \Rightarrow p = q$$

Solving the equations simultaneously for p and q ,

$$p + 3 = 2p \Rightarrow p = 3$$

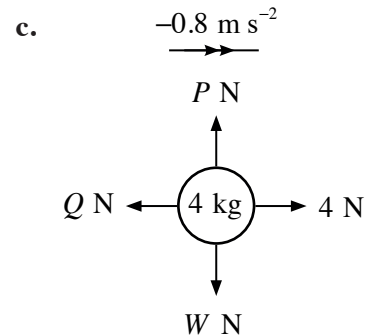
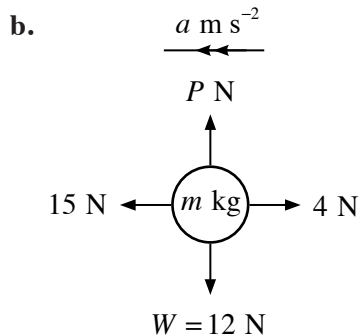
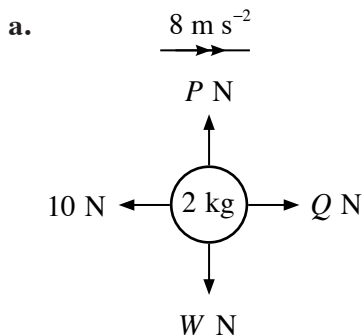
$$\Rightarrow q = 3$$

$$\therefore p = q = 3$$

Use N2 and resolve vertically and horizontally. Remember, $a = 0$ in each case.

Exercise 10A

- 1 A particle of weight 19.6 N accelerates at 4 m/s^2 to the right. Find the magnitude and direction of the resultant force on the particle.
- 2 A book of mass m kg is at rest on a smooth table. The normal reaction force exerted by the table on the book is 58.8 N. Find the value of m .
- 3 A body of mass 30 kg is falling freely under the influence of gravity. The force due to air resistance is 50 N.
 - a. Draw a force diagram, showing all the forces acting on the body.
 - b. Find the acceleration of the body.
- 4 In each of the cases, below find the missing values. (Take W = weight of the body.)



- 5 A particle of mass 3 kg moves horizontally at constant speed on a rough horizontal surface. The magnitude of the force that causes the particle to move is equal to the magnitude of the particle's normal reaction force. Find the force due to friction between the particle and the surface.
- 6 A particle of mass 2 kg is moving at a constant speed of 20 m/s on a smooth horizontal surface. The surface then becomes rough and the particle is subject to a frictional force of 25 N. The particle subsequently comes to rest. Find the time taken for the particle to come to rest.

As the particle moves through the liquid, a simple model involves the particle experiencing a constant resistance to motion of kD N, where k is the mass of the liquid in the container and D is a constant.

b. Show that the distance travelled by the particle before coming to rest, x m, is given by

$$x = h \left(1 + \frac{mg}{\rho VD - mg} \right)$$

Key points from Chapter 10

- 1** Newton's 1st Law states that every body will continue in its state of rest or uniform motion unless acted upon by an external force.
- 2** Newton's 2nd Law states that the resultant force acting on a body is equal to the mass of the body multiplied by the body's acceleration in the direction of the resultant force. Mathematically, in scalar form, we write $F = ma$.
- 3** Newton's 3rd Law states that to every action, there is an equal and opposite reaction.
- 4** The weight of an object is the force that acts on it due to gravity. If an object has mass m kg, its weight, W N, is $W = mg$, where g is the acceleration due to gravity. The value of g depends on location - for example, it is different on the Moon compared to Earth and different at different heights above the Earth's surface. You should assume that $g = 9.8 \text{ m s}^{-2}$ unless told otherwise.
- 5** When an object is in contact with a surface, the surface will exert a reaction force that is perpendicular/normal to the surface. This force is called the normal reaction.
- 6** The frictional force between an object and a surface is the force that opposes the motion of the object.
- 7** In vector form, Newton's 2nd law is $\mathbf{F} = m\mathbf{a}$.
- 8** If a set of particles are connected and the system travels in the same straight line, then the system can be considered as a single particle.
- 9** Tension in a string/rod acts inwards, pulling particles together.
- 10** A string cannot be in thrust. However, a rod can be in thrust, in which case the forces act outwards from the centre of the rod.