SCAN ME

crashMATHS

PROOF BY INDUCTION WORKSHEET

crashmathsworksheets

1 Prove by induction that, for $n \in \mathbf{Z}^+$,

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(2n+1)(n+1)$$

Question 1 continued

2 Show by mathematical induction that

$$6^n - 1$$

is divisible by 5 for $n \in \mathbf{Z}^+$.

Question 2 continued

3 A sequence of terms u_1, u_2, u_3 ..., is defined by

$$u_{n+1} = 5u_n - 8$$

Given that $u_1 = 3$,

Prove by mathematical induction that $u_n = 5^{n-1} + 2$, for $n \in \mathbf{Z}^+$.

Question 3 continued

4 Using the method of mathematical induction, prove that

$$\sum_{r=1}^{n} r = \frac{n}{2}(1+n)$$

for $n \in \mathbf{Z}^+$.

Question 4 continued

Do not write outside the box

5 Prove, using mathematical induction, that for $n \in \mathbf{Z}^+$

$$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^n = \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix}$$

Question 5 continued

6 Using the method of mathematical induction, prove that

$$1+2+4+8+...+2^{n-1}=2^n-1$$

for all positive integers n.

Do not write outside the box

Question 6 continued

7 Given that

$$5^n + 2\left(11^n\right)$$

is a multiple of 3 for all positive integers n, use mathematical induction to prove this result.

Do not write outside the box

Question 7 continued

8 (a) Use mathematical induction to prove that, for all positive integers n,

$$\sum_{r=1}^{n} r^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

(b) Hence, show that

$$\sum_{r=1}^{n} \left(r^4 - r^2 - r\right) = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1)$$

Do not write outside the box

Question 8 continued

9 A recurrence relationship is defined such that

$$u_1 = 2,\ u_2 = 6$$

$$u_{n+3} = 6u_{n+2} - 5u_{n+1}\ ,$$

Use mathematical induction to prove that, for $n \in \mathbf{Z}^+$, $u_n = 5^{n-1} + 1$.

Do not write outside the box

Question 9 continued

10 Show, by mathematical induction that,

$$\sum_{r=1}^{m}(2r-1)=m^2$$

for $m \in \mathbf{Z}^+$.

Do not write outside the box

Question 10 continued

11 Prove, using mathematical induction, that

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{pmatrix}$$

for all positive integers n.

Question 11 continued

12 For all positive integers n, use the method of mathematical induction to prove

that

$$\left(1-\frac{1}{n^2}\right)! = \frac{n+1}{2n}$$

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Question 12 continued

13 Using mathematical induction, prove that

$$3^{2n+3} + 40n - 27$$

is divisible by 64 for $n \in \mathbf{Z}^+$.

Question 13 continued

14 It is given that

$$\begin{pmatrix} 1 & c \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & (2^n - 1)c \\ 0 & 2^n \end{pmatrix}$$

where c is a constant.

Using induction, prove this result for all positive integers n.

Question 14 continued

15 Prove, using mathematical induction, that for $n \in \mathbf{Z}^+$

$$n^3 - 7n + 3 \equiv 3Q$$

for some polynomial Q.

Question 15 continued

16 Use the method of mathematical induction to prove that, if $n \in \mathbf{Z}^+$,

$$\sum_{r=1}^{n}\left(\frac{1}{\sqrt{r-1}+\sqrt{r}}\right)=\sqrt{n}-1$$

Do not write outside the box

Question 16 continued