

crashMATHS -

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1	(a) Find the value of	$\sum_{r=1}^{10} r^2$
	(b) Find the value of	$\sum_{r=1}^{10} r^3$
	(c) Hence, or otherwise, find the value of	$\sum_{r=10}^{20} \left(r^3 - r^2 \right)$
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(b) Using the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} (2r-3)^3 = n \left(an^3 + bn^2 + cn + d \right)$$

where a, b, c and d are constants to be found.

(c) Find the value of

$$\sum_{r=14}^{96} (2r-3)^3$$

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Question 2 continued		





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3 Show that, using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,
$\sum_{r=1}^{n} r(3-r) = -\frac{1}{3}n(n-4)(n+1)$
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Question 3 continued			





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4 Using the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$, show that
$\sum_{r=1}^{n} \left(5r^3 - 4r + 1 \right) = \frac{1}{4} n \left[5n^2 \left(n + 2 \right) - 3\left(n + 4 \right) \right]$

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Question 4 continued		





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5	(a) Show that, using standard results for	$\sum_{n=0}^{\infty} r$,	$\sum_{n=0}^{\infty} r^2$ and	$\sum_{n=0}^{\infty} r^3$,
		r=1	r=1	r=1

$$\sum_{r=1}^{n} r(r+3)(r+12) = \frac{1}{4}n(n+1)(n^2+21n+82)$$

(b) Hence, show that the value of

$$\sum_{r=20}^{128} r(r+3)(r+12) = 78987722$$

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Question 5 continued			





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6	(2)	Express
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$$1^2 + 2^2 + 3^2 \dots + 123^2$$

in the form $\sum_{n=x}^{y} f(n)$, where x and y are constants to be found.

(b) Hence, or otherwise, find the value

$$1^2 + 2^2 + 3^2 \dots + 123^2$$

(c) By the use of a similar method, or otherwise, find the sum of the series

$$2^2 + 6^2 + 10^2 + ... + 90^2 + 94^2 + 98^2$$

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7 Show clearly that $\sum_{r=1}^{3n} \left(1 - 5r - r^2 + 2^r\right) = -9n^3 - 27n^2 - 5n + 2\left(8^n - 1\right)$



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8	(a) Show that, using the standard results for	$\sum r$,	$\sum r^2$ and	$\sum r^3$,
		r=1	r=1	r=1

$$\sum_{r=1}^{n} r(2r+3)(2r-3) = \frac{1}{2}n(n+1)(an^{2}+bn+c)$$

where a, b and c are constants to be found.

(b) Hence, find the exact value of

$$\sum_{r=3}^{50} r(2r+3)(2r-3) - \sum_{r=5}^{25} r(2r+3)(2r-3)$$

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Question 8 continued			





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9	(a) Using standard results for	$\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that
		$\sum_{r=1}^{n} r(r+12) = \frac{1}{6} n(n+1)(2n+37)$

(b) Now, using the method of mathematical induction, prove that

$$\sum_{r=1}^{n} r(r+12) = \frac{1}{6} n(n+1)(2n+37)$$

for all $n \in \mathbb{Z}^+$

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10	(a)	Prove	that,	for	$n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} r^{5} = \frac{1}{12} n^{2} (n+1)^{2} (2n^{2} + 2n - 1)$$

(b) Hence, by using the standard results for $\sum_{r=1}^{n} r^5$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$, show that

$$\sum_{r=1}^{n} \left(r^5 - 12r^2 + 24r \right) = \frac{1}{12} n \left(n + 1 \right) \left(2n^4 + 4n^3 + n^2 - 49n + 120 \right)$$

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Question 10 continued			





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11 Using standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, find an expression for $\sum_{r=1}^{n} \left(\sin k \cdot r^2 - \cos k \right)$
where k is a constant such that $-1 \le k \le 1$.



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Question 11 continued			





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12 (a) Show, using standard formulae, that $\sum_{r=1}^{n} (5r+2) = \frac{1}{2} n (5n+1)$
(b) Hence, find an expression for $\sum_{r=1}^{n} \left(5r^2 - 2 \sum_{r=n}^{3n} (5r+2) \right)$
Give your answer in its simplest form.

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13 Find the values of x such that			
$6\sum_{r=1}^{x} (2r - r^2) = 10 - x^2$			
for all $x > 0$.			

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$$\sum_{r=1}^{n} \left(r^3 + 2f(r) \right) = \frac{1}{12} n \left(3n^3 + an^2 + bn + 58 \right)$$

and

$$f'(r) = \frac{ar}{2} + \frac{b}{17}$$

- (i) Find a and b.
- (ii) Find f(r).

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