



crash**MATHS**

SERIES
WORKSHEET



crashmathsworksheets

2 (a) Expand $(2r - 3)^3$.

(b) Using the standard results for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$, show that

$$\sum_{r=1}^n (2r - 3)^3 = n(an^3 + bn^2 + cn + d)$$

where a , b , c and d are constants to be found.

(c) Find the value of

$$\sum_{r=14}^{96} (2r - 3)^3$$



3 Show that, using the standard results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$,

$$\sum_{r=1}^n r(3-r) = -\frac{1}{3}n(n-4)(n+1)$$



4 Using the standard results for $\sum_{r=1}^n r^3$ and $\sum_{r=1}^n r$, show that

$$\sum_{r=1}^n (5r^3 - 4r + 1) = \frac{1}{4}n[5n^2(n+2) - 3(n+4)]$$



- 5 (a) Show that, using standard results for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$,

$$\sum_{r=1}^n r(r+3)(r+12) = \frac{1}{4}n(n+1)(n^2 + 21n + 82)$$

- (b) Hence, show that the value of

$$\sum_{r=20}^{128} r(r+3)(r+12) = 78987722$$



6 (a) Express

$$1^2 + 2^2 + 3^2 \dots + 123^2$$

in the form $\sum_{n=x}^y f(n)$, where x and y are constants to be found.

(b) Hence, or otherwise, find the value

$$1^2 + 2^2 + 3^2 \dots + 123^2$$

(c) By the use of a similar method, or otherwise, find the sum of the series

$$2^2 + 6^2 + 10^2 + \dots + 90^2 + 94^2 + 98^2$$



7 Show clearly that

$$\sum_{r=1}^{3n} (1 - 5r - r^2 + 2^r) = -9n^3 - 27n^2 - 5n + 2(8^n - 1)$$



- 8 (a) Show that, using the standard results for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$,

$$\sum_{r=1}^n r(2r+3)(2r-3) = \frac{1}{2}n(n+1)(an^2 + bn + c)$$

where a , b and c are constants to be found.

- (b) Hence, find the exact value of

$$\sum_{r=3}^{50} r(2r+3)(2r-3) - \sum_{r=5}^{25} r(2r+3)(2r-3)$$





Question 8 continued

Handwriting practice area consisting of 21 horizontal lines.



10 (a) Prove that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^n r^5 = \frac{1}{12} n^2 (n+1)^2 (2n^2 + 2n - 1)$$

(b) Hence, by using the standard results for $\sum_{r=1}^n r^5$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r$, show that

$$\sum_{r=1}^n (r^5 - 12r^2 + 24r) = \frac{1}{12} n(n+1)(2n^4 + 4n^3 + n^2 - 49n + 120)$$



12 (a) Show, using standard formulae, that

$$\sum_{r=1}^n (5r+2) = \frac{1}{2}n(5n+1)$$

(b) Hence, find an expression for

$$\sum_{r=1}^n \left(5r^2 - 2 \sum_{r=n}^{3n} (5r+2) \right)$$

Give your answer in its simplest form.



Extra space



A series of horizontal lines for writing, providing extra space for the user's work.



