SCAN ME

crashMATHS

NUMERICAL METHODS WORKSHEET

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1 A function f is defined such that

$$f(x) = x^3 + x^2 - 6$$

(a) Show that $f(x) = 0$ has a real root α within the interval $[1, 2]$.

(b) Use interval bisection twice to obtain an interval of width 0.25 that contains α.

(c) Starting with $x_0 = 1$, use the Newton-Raphson process on $f(x)$ twice to find an approximation for α, giving your answer to three significant figures.

Question 1 continued

2 Given that $f(x) = x^3 + 2x^2 + 4x - 6$ has a root within the interval $[0.5, 1]$, use linear interpolation once to find an approximation for this root.

Question 2 continued

3 The curve

$$y = x^2 + \frac{5\sqrt{x} - x^3}{x} - \frac{7}{6}$$

has one root α.

Starting with $x_0 = 18.8$, use the Newton-Raphson process twice to find an approximation for α, giving your answer to five decimal places.

Question 3 continued

4 (a) Show that the equation $\tan x - 3 = 0$ has a root λ within the interval $[7.4, 7.8]$.

(b) Use interval bisection to find an interval in the form $[a_1, a_2]$ containing λ, such that $a_2 - a_1 = 0.1$.

(c) Using the interval $[a_1, a_2]$, use linear interpolation once to find an approximation for λ to two decimal places.

Question 4 continued

5

$$f(x) = 2^x - x^2 - 1$$

The equation $f(x) = 0$ has a root α in the interval $[4.256, 4.26]$.

(a) Use interval bisection three times to find an interval of width 5×10^{-4} that contains α.

(b) State the value of α to three decimal places.

Question 5 continued

6 Given that

$$f(x) = \frac{5x^3 - 2x - 3}{\sqrt{x}}$$

(a) Find $f'(x)$.

(b) Show that $f(x) = 0$ has only one real root.

(c) Show that the real root of $f(x) = 0$ lies between 0.5 and 1.5.

(d) Starting with $x = 0.5$, use the Newton-Raphson method twice to obtain an approximation for the real root of $f(x) = 0$. Give your answer to three decimal places.

(e) Deduce whether the approximation you found in part (d) is accurate to three decimal places.

Question 6 continued

7 The curve $y = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$ has a root between $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$.

Use linear interpolation once on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ to find an approximation for this root. Give your answer correct to two significant figures.

Question 7 continued

8 The curve $y = f(t)$ is defined such that

$$f(t) = t^3 + 3t^2 + 2t - 4$$

(a) Work out the stationary points on the curve $y = f(t)$.

(b) Hence, by sketching, show that the curve has one real root.

(c) Starting with $x_0 = 1$, use the Newton-Raphson method once to find an approximation for this real root.

Question 8 continued

9 (a) Show that when the Newton-Raphson method is used on the equation

$$x^2 - a = 0$$

with $x_n = \beta$, the resultant iterative function is

$$x_{n+1} = \frac{1}{2}\left(\beta + \frac{a}{\beta}\right)$$

(b) Hence, given the equation

$$x^2 - 7 = 0$$

Use the Newton-Raphson method once to find x_1, given that $x_0 = \frac{5}{2}$.

Question 9 continued

[This is an extension question and is included for stretch and challenge purposes only.]

10 This question will investigate the number of iterations of interval bisection needed to converge to a root within a certain error of tolerance, ε.

Let the function $y = f(x)$ have a root α within the interval $[a_0, b_0]$.

If N denotes the minimum number of iterations of interval bisection needed to obtain an approximation of α within an error of tolerance ε, explain why

$$\frac{b_0 - a_0}{2^N} \leq \varepsilon$$

Deduce the number of iterations to obtain an approximation of the root α with an error of tolerance of 10^{-3}, if $a_0 = 0$ and $b_0 = 1$.

Question 10 continued