SCAN ME

crash**MATHS**

COORDINATE SYSTEMS WORKSHEET

crashmathsworksheets

1 A parabola has the equation $x = 5t^2$ and $y = 10t$.

(a) Find the Cartesian equation for this parabola.

(b) State the focus of the parabola.

(c) Work out the equation of the directrix of the parabola.

Question 1 continued

2 The focus of a parabola is $(2a+3,0)$.

The parabola has Cartesian equation $y^2 = 12x$.

Find the value of a.

Question 2 continued

3 In the space below, on separate axis, sketch the graphs of

(a) $y^2 = 4ax$

(b) $y = 4ax^2$

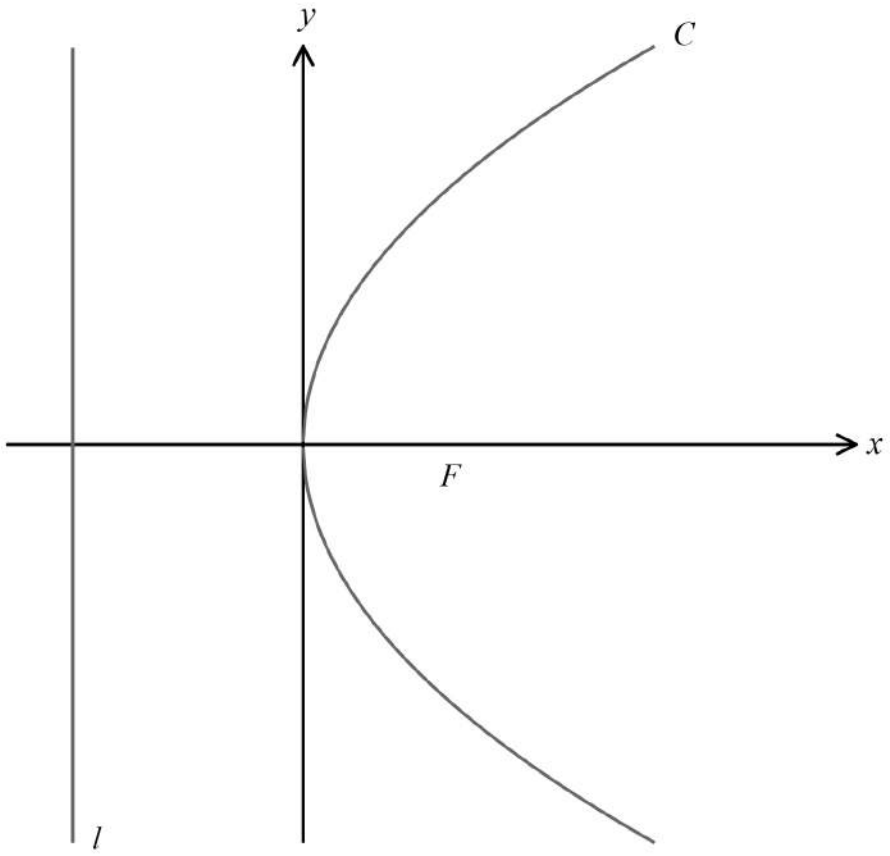
On your sketches, you should show clearly the position and coordinates of the foci and the position and equation of the directrices of these parabolas.

Question 3 continued

4 Find the equation of the tangent to the rectangular hyperbola with equation $y = \frac{c}{x}$ at the point $x = c^2$.

Question 4 continued

5 A parabola C has the Cartesian equation $y^2 = 36x$.



F is the focus to C.

(a) State the coordinates of F.

The line l is the directrix of C.

(b) Find the equation of l.

The points P and Q are both at a distance of 12 units away from the directrix of the parabola.

(c) Find the exact length of PQ, giving your answer as a surd in its simplest form.

The point X has coordinates $(x, 6\sqrt{3})$.

(d) Work out the area of the triangle PQX.

Question 5 continued

6 (a) Show that the equation of the tangent to the parabola at the point $P\ (ap^2, 2ap)$ is

$$py - ap^2 = x$$

(b) State the equation of the tangent to the parabola at the point $Q\ (aq^4, 2aq^2)$.

The tangents at the points P and Q intersect at the point R.

(c) Find, in their simplest form, the coordinates of R.

Given that R also lies on the line with equation $x = 2a$,

(d) Find p in terms of q.

Question 6 continued

7 The parabola P has focus $(3, 0)$.

The rectangular hyperbola R has parametric equations $x = 3t\sqrt{2}$ and $y = \dfrac{3\sqrt{2}}{t}$.

P and R intersect at the point M.

Find the coordinates of M.

Question 7 continued

8 The point $P\ (at^2, 2at)$ lies on a parabola C with equation $y^2 = 4ax$, where $a > 0$.

(a) Show that an equation of the normal to C at P is

$$y + tx = t(at^2 + 2a)$$

Given that the point $R\left(a, \frac{a}{2}\right)$ lies on the normal to C at P,

(b) Find the value of t.

Question 8 continued

9 The points $M\ (8,8)$ and $N\ (18,b)$, where $b>0$, lie on the parabola C with equation $y^2=4ax$.

(a) Find ab.

Given that F is the focus of C,

(b) Calculate the area of the triangle MNF.

Question 9 continued

10 The point $M(6t^2, 12t)$ lies on the curve C with equation $12x - y^2 = 0$.

(a) Show that the equation of the tangent to C at the point M is

$$ty - x = 6t^2$$

The tangent to the point M on C passes through the point $A\ (-6, 4)$.

(b) Find the possible values of t.

Question 10 continued

11 The point P lies on the rectangular hyperbola H with equation $xy = c^2$, where $c < 0$.

(a) Find the equation of the tangent to H at the point $\left(ct, \frac{c}{t}\right)$.

The tangent crosses the x axis at the point P and the y axis at the point Q.

(b) In terms of c and t, work out the coordinates of P and Q.

Given that O is the origin and that the area of the triangle OPQ is 120,

(c) Find the exact value of c.

Question 11 continued

12 The point $P\ (at^2, 2at)$, where $t > 0$, lies on the parabola with equation $y^2 = 4ax$.

The tangent and normal at P cut the y axis at the points A and B respectively.

Find the value of $\dfrac{|PA|}{|PB|}$.

Question 12 continued

13 The rectangular hyperbola H has Cartesian equation $xy = 8$.

The point $P\left(8p, \frac{8}{p}\right)$ and the point $Q\left(8q, \frac{8}{q}\right)$ lie on H, where $p, q \neq 0$ and $p \neq q$.

The tangents to H at P and Q meet at the point N.

(a) Find the coordinates of N.

(b) Find the value of p^2q^2 when the line joining N to the origin is perpendicular to the chord PQ.

Question 13 continued

14 The normal to the parabola $y^2 = 4ax$ intersects the parabola at the point $P\left(ap^2, 2ap\right)$.

The normal to the parabola at P then meets the curve again at another point Q.

(a) Show that the coordinates of Q are

$$\left(\frac{a\left(p^2+2\right)^2}{p^2}, -\frac{-2a\left(p^2+2\right)}{p}\right)$$

The tangents to the parabola at P and Q intersect at a point R.

(b) Find the coordinates of R.

(c) Show that the locus of R is

$$y^2\left(x+2a\right)+4a^3=0$$

Question 14 continued

15

$$C_1 : x^2 = 4ay$$

$$C_2 : y^2 = 4ax$$

The parabolas C_1 and C_2 intersect at the origin and at a point P. The tangent to C_1 at the point P intersects C_2 at the point A.

(a) Find the coordinates of A.

The tangent to C_2 at the point P intersects C_1 at the point B.

The angle $APB = x^\circ$

(b) Show that $\tan x = \frac{3}{4}$.

(c) Show also that the line AB is a common tangent to both C_1 and C_2.

Question 15 continued

16 The parabola C has equation $y^2 = 15x$.

(a) Prove that the line $y = mx + \frac{15}{4m}$ is a tangent to C, for all non-zero values of m.

(b) Hence, or otherwise, find the equations of the common tangents to C and the circle $x^2 + y^2 = 16$.

Question 16 continued

Extra space

Extra space

Extra space